

ECE 312 (SP26):

# Sorting Algorithms

Neil Zhao

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# Bubble Sort Naïve

```
void bubble_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
            }  
        }  
    }  
}
```

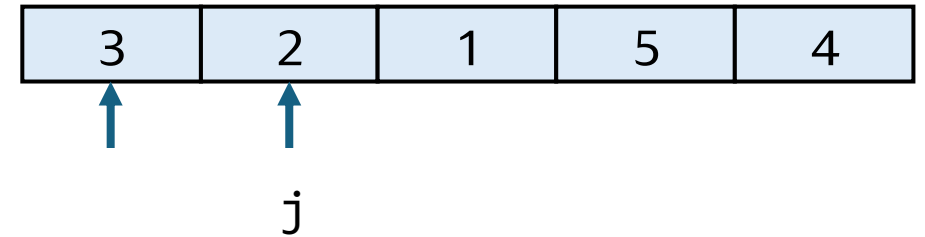


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    if (sz <= 1) return;
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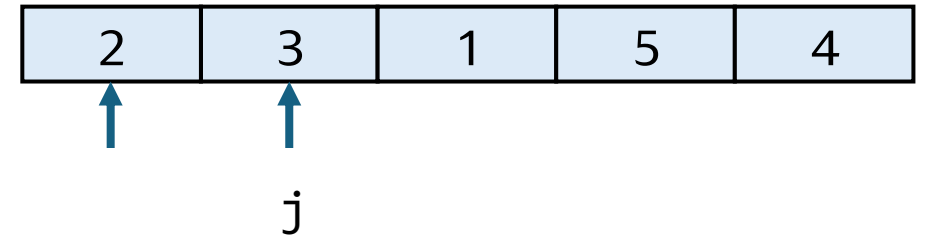
```
            }  
        }  
    }  
}
```



$i = 0; j = 1$

# Bubble Sort Naïve

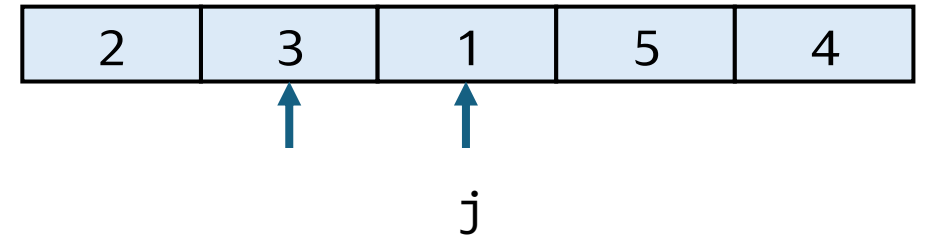
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            }  
        }  
    }  
}
```



$i = 0; j = 1$

# Bubble Sort Naïve

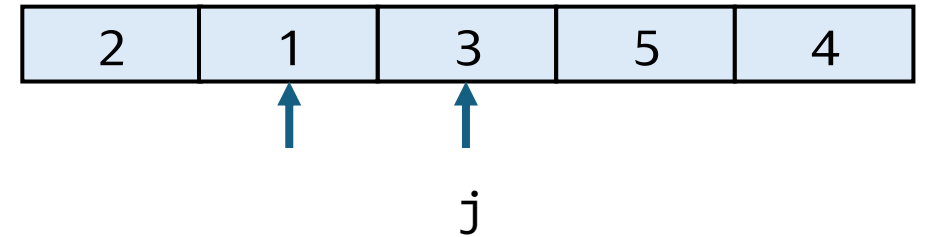
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void bubble_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
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            }  
        }  
    }  
}
```



$i = 0; j = 2$

# Bubble Sort Naïve

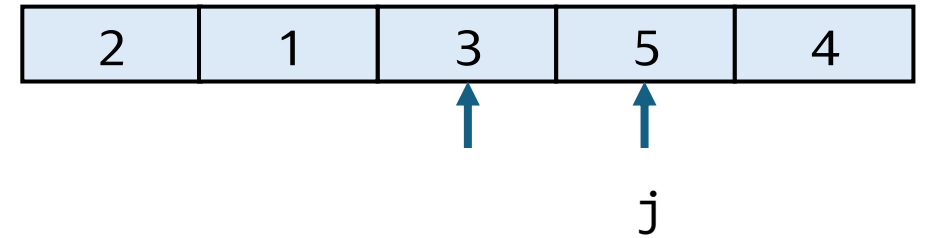
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    for (size_t i = 0; i < sz - 1; i++) {  
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            if (arr[j] < arr[j - 1]) {  
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            }  
        }  
    }  
}
```



$i = 0; j = 2$

# Bubble Sort Naïve

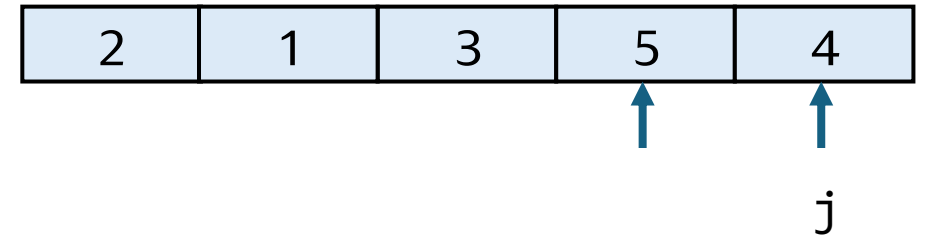
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            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
            }  
        }  
    }  
}
```



$i = 0; j = 3$

# Bubble Sort Naïve

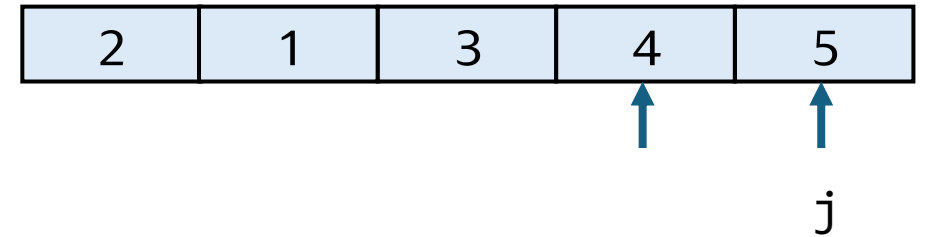
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    for (size_t i = 0; i < sz - 1; i++) {  
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            if (arr[j] < arr[j - 1]) {  
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    }  
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$i = 0; j = 4$

# Bubble Sort Naïve

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void bubble_sort_v0(int *arr, size_t sz) {  
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        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
            }  
        }  
    }  
}
```

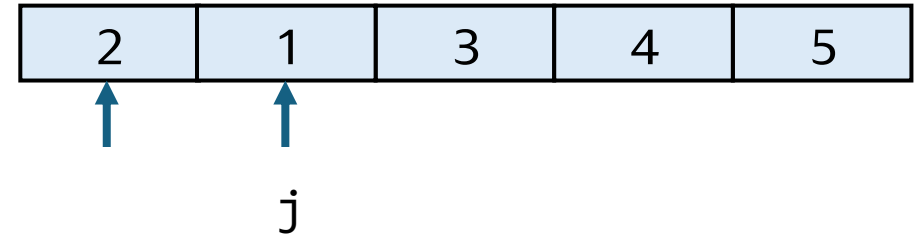


$i = 0; j = 4$

# Bubble Sort Naïve

```
void bubble_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;
```

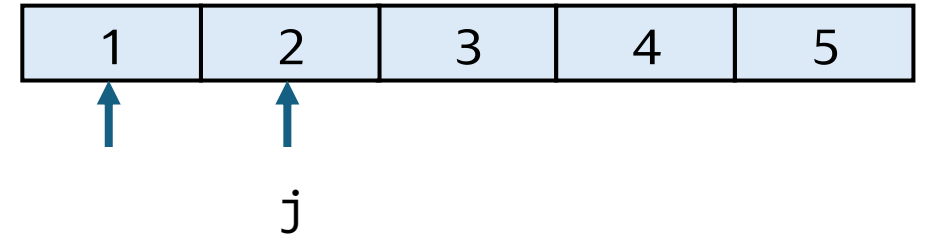
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    for (size_t i = 0; i < sz - 1; i++) {  
        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
            }  
        }  
    }  
}
```



$i = 1; j = 1$

# Bubble Sort Naïve

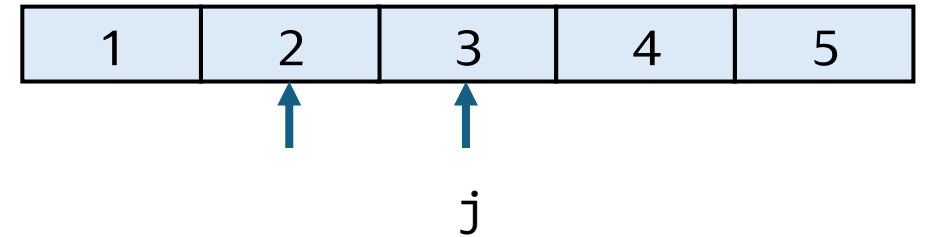
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$i = 1; j = 1$

# Bubble Sort Naïve

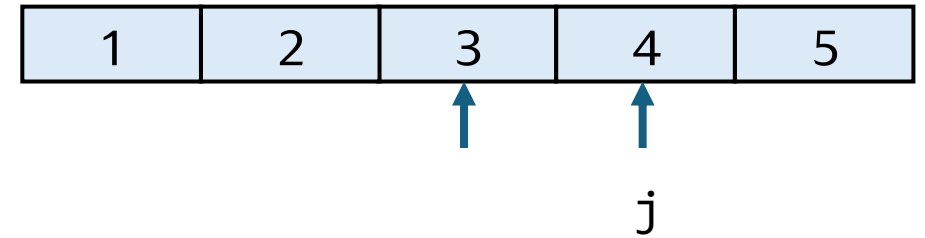
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                swap_int(&arr[j - 1], &arr[j]);  
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        }  
    }  
}
```



$i = 1; j = 2$

# Bubble Sort Naïve

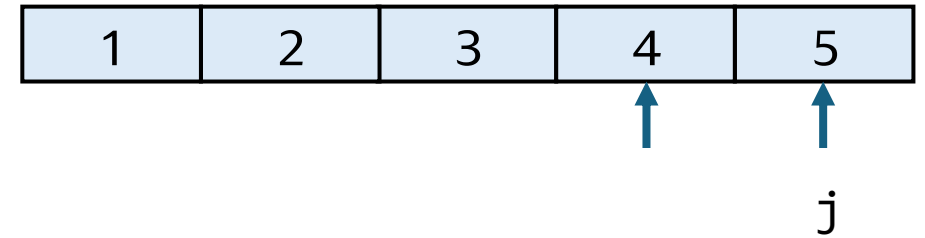
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void bubble_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
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            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
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        }  
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```



$i = 1; j = 3$

# Bubble Sort Naïve

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    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
            }  
        }  
    }  
}
```



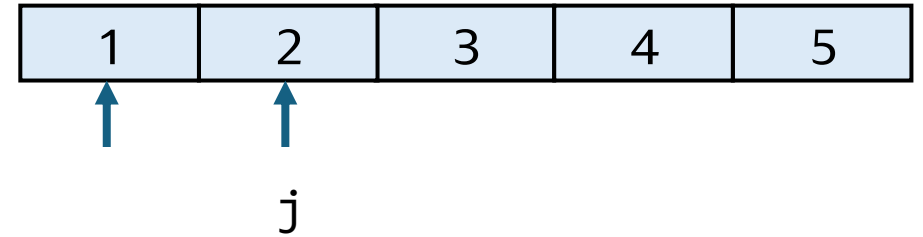
$i = 1; j = 4$

# Bubble Sort Naïve

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void bubble_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;
```

```
    for (size_t i = 0; i < sz - 1; i++) {  
        for (size_t j = 1; j < sz; j++) {  
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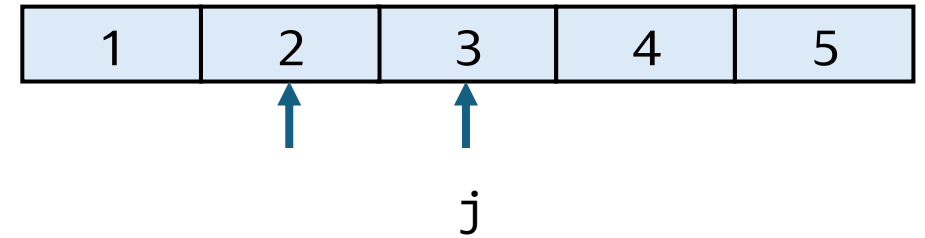
```
            }  
        }  
    }  
}
```



$i = 2; j = 1$

# Bubble Sort Naïve

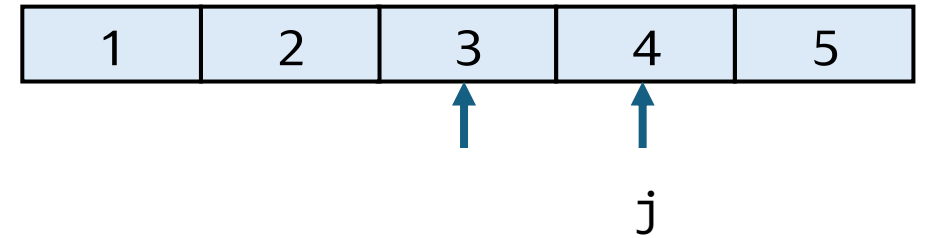
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void bubble_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
            }  
        }  
    }  
}
```



$i = 2; j = 2$

# Bubble Sort Naïve

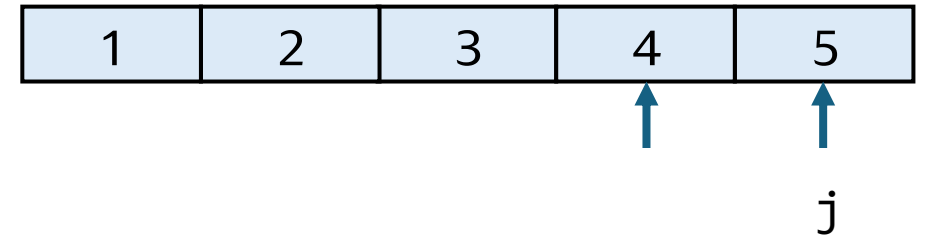
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            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
            }  
        }  
    }  
}
```



$i = 2; j = 3$

# Bubble Sort Naïve

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void bubble_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
            }  
        }  
    }  
}
```

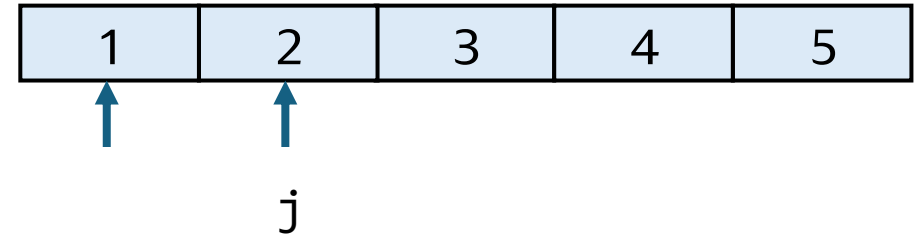


$i = 2; j = 4$

# Bubble Sort Naïve

```
void bubble_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;
```

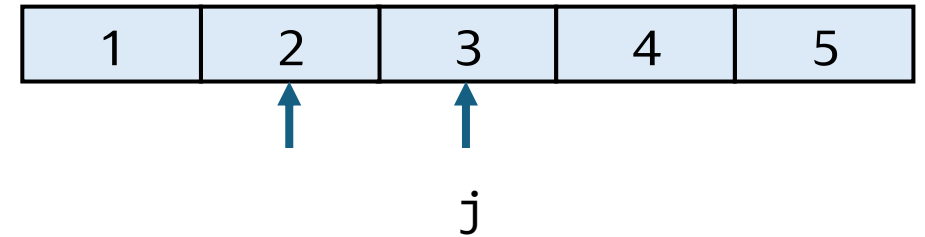
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    for (size_t i = 0; i < sz - 1; i++) {  
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            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
            }  
        }  
    }  
}
```



$i = 3; j = 1$

# Bubble Sort Naïve

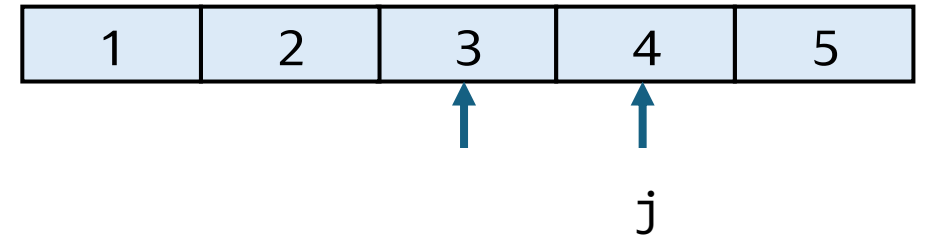
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            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
            }  
        }  
    }  
}
```



$i = 3; j = 2$

# Bubble Sort Naïve

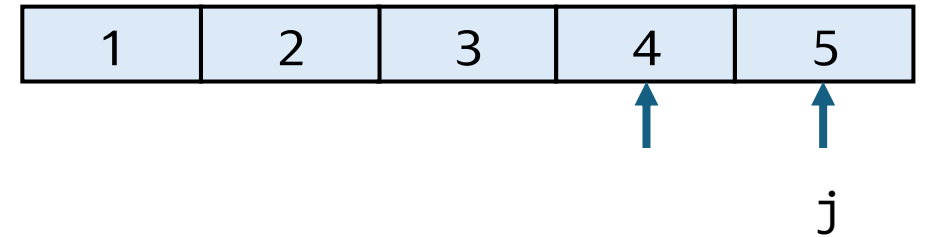
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            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
            }  
        }  
    }  
}
```



$i = 3; j = 3$

# Bubble Sort Naïve

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void bubble_sort_v0(int *arr, size_t sz) {  
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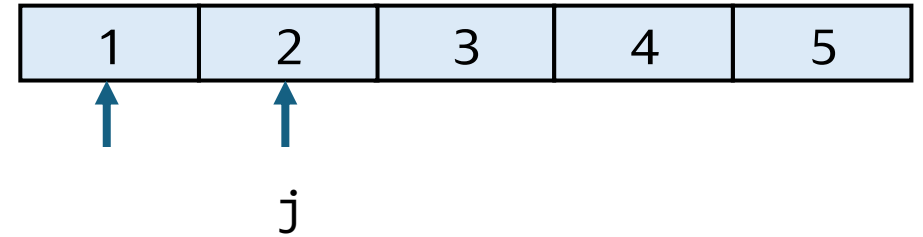


$i = 3; j = 4$

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    if (sz <= 1) return;
```

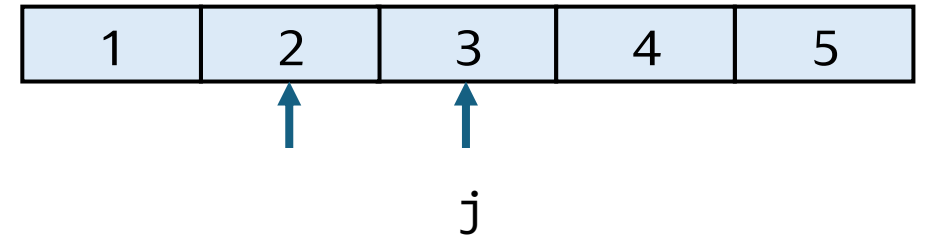
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$i = 4; j = 1$

# Bubble Sort Naïve

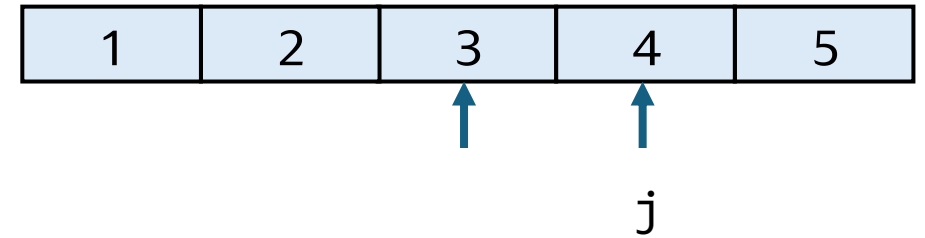
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$i = 4; j = 2$

# Bubble Sort Naïve

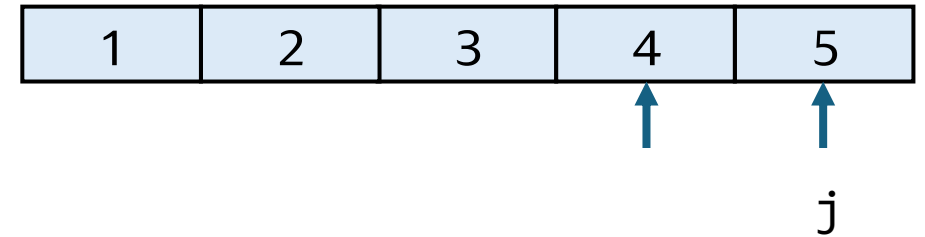
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    }  
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$i = 4; j = 3$

# Bubble Sort Naïve

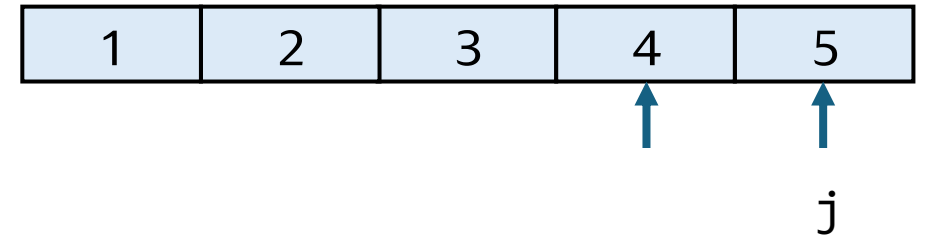
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        }  
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```



$i = 4; j = 4$

# Bubble Sort Naïve

```
void bubble_sort_v0(int *arr, size_t sz) {  
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    for (size_t i = 0; i < sz - 1; i++) {  
        for (size_t j = 1; j < sz; j++) {  
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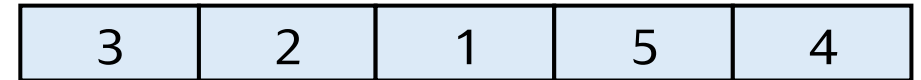
Worst-case:  $(n - 1)^2$  comparisons +  $n(n - 1)/2$  swaps  
Best-case:  $(n - 1)^2$  comparisons

# Bubble Sort with Early Termination

```
void bubble_sort_v1(int *arr, size_t sz) {
    if (sz <= 1) return;

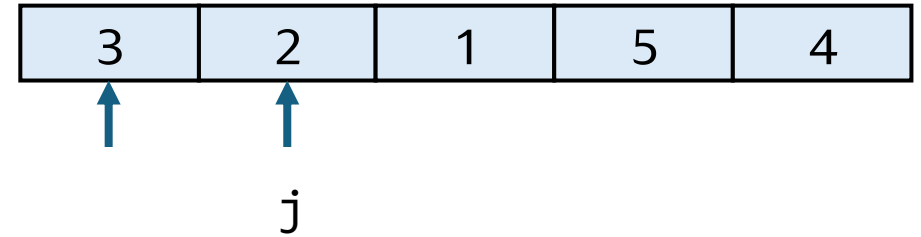
    for (size_t i = 0; i < sz - 1; i++) {
        bool swapped = false;
        for (size_t j = 1; j < sz; j++) {
            if (arr[j] < arr[j - 1]) {
                swap_int(&arr[j - 1], &arr[j]);
                swapped = true;
            }
        }

        if (!swapped) break;
    }
}
```



# Bubble Sort with Early Termination

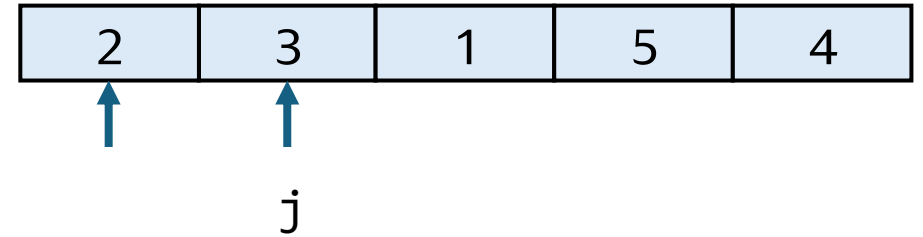
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            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



`i = 0; j = 1; swapped = false`

# Bubble Sort with Early Termination

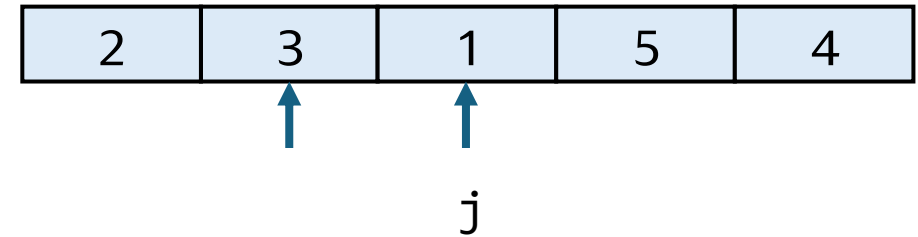
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}
```



`i = 0; j = 1; swapped = true`

# Bubble Sort with Early Termination

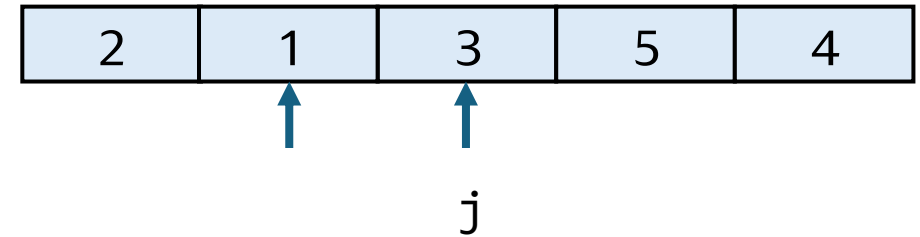
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                swapped = true;  
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        }  
  
        if (!swapped) break;  
    }  
}
```



`i = 0; j = 2; swapped = true`

# Bubble Sort with Early Termination

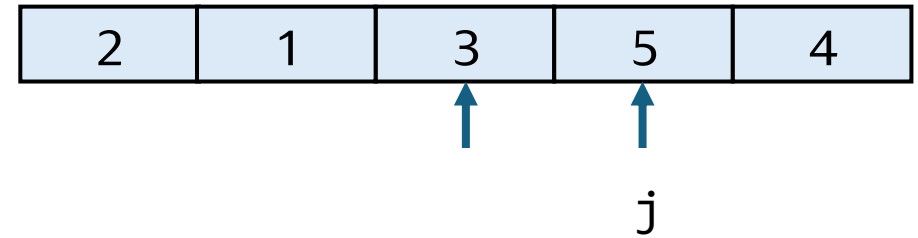
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        if (!swapped) break;  
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}
```



`i = 0; j = 2; swapped = true`

# Bubble Sort with Early Termination

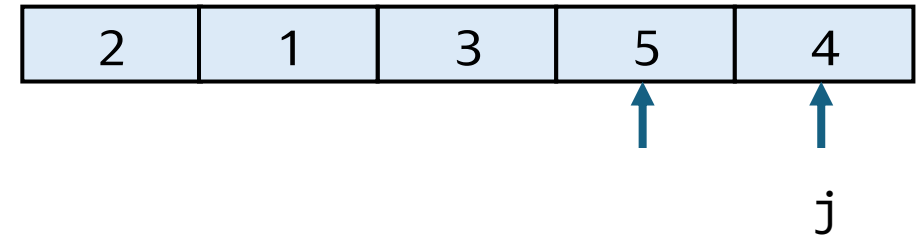
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            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



`i = 0; j = 3; swapped = true`

# Bubble Sort with Early Termination

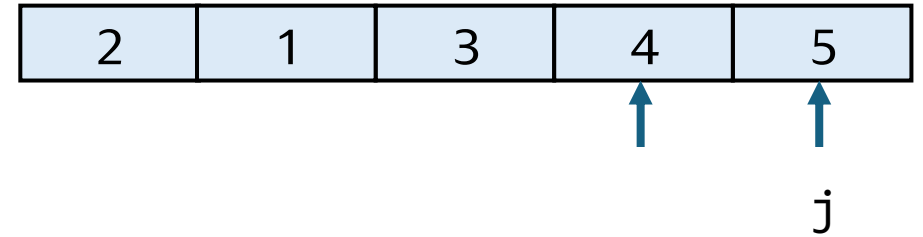
```
void bubble_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



`i = 0; j = 4; swapped = true`

# Bubble Sort with Early Termination

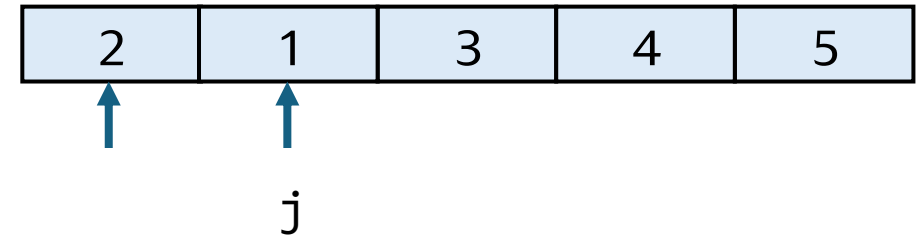
```
void bubble_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



`i = 0; j = 4; swapped = true`

# Bubble Sort with Early Termination

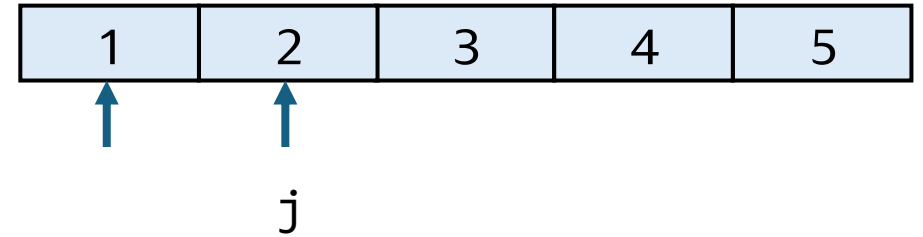
```
void bubble_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



`i = 1; j = 1; swapped = false`

# Bubble Sort with Early Termination

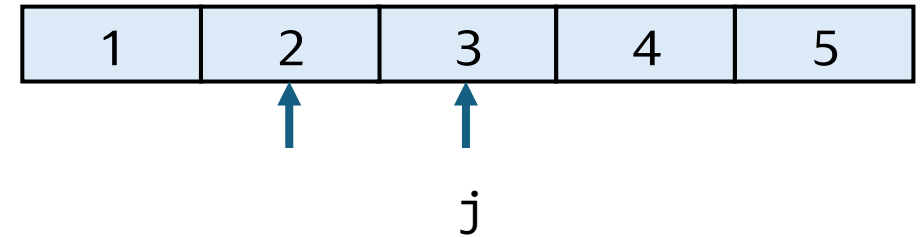
```
void bubble_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



`i = 1; j = 1; swapped = true`

# Bubble Sort with Early Termination

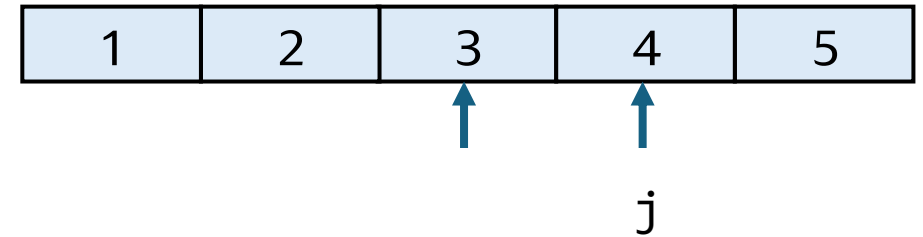
```
void bubble_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



`i = 1; j = 2; swapped = true`

# Bubble Sort with Early Termination

```
void bubble_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



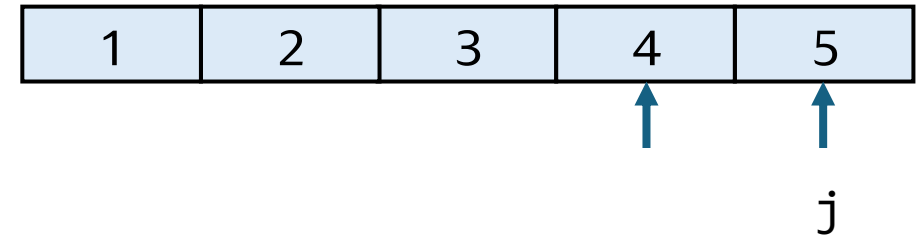
`i = 1; j = 3; swapped = true`

# Bubble Sort with Early Termination

```
void bubble_sort_v1(int *arr, size_t sz) {
    if (sz <= 1) return;

    for (size_t i = 0; i < sz - 1; i++) {
        bool swapped = false;
        for (size_t j = 1; j < sz; j++) {
            if (arr[j] < arr[j - 1]) {
                swap_int(&arr[j - 1], &arr[j]);
                swapped = true;
            }
        }

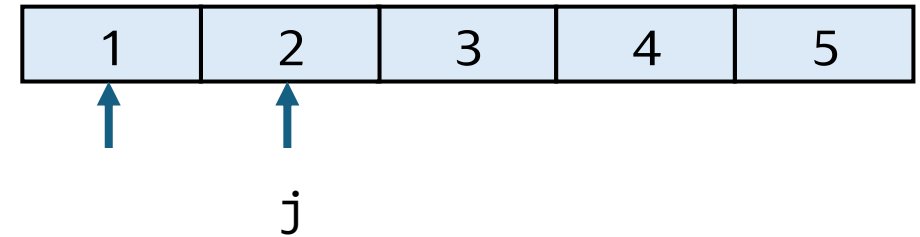
        if (!swapped) break;
    }
}
```



`i = 1; j = 4; swapped = true`

# Bubble Sort with Early Termination

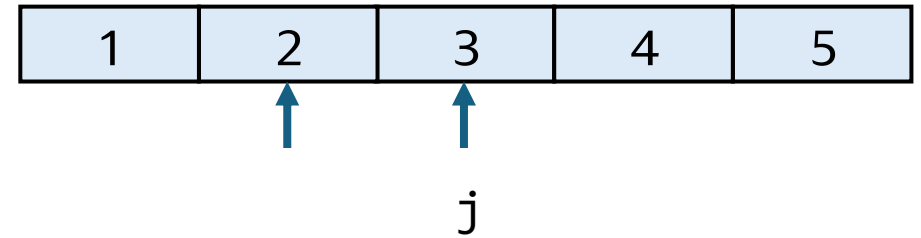
```
void bubble_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



`i = 2; j = 1; swapped = false`

# Bubble Sort with Early Termination

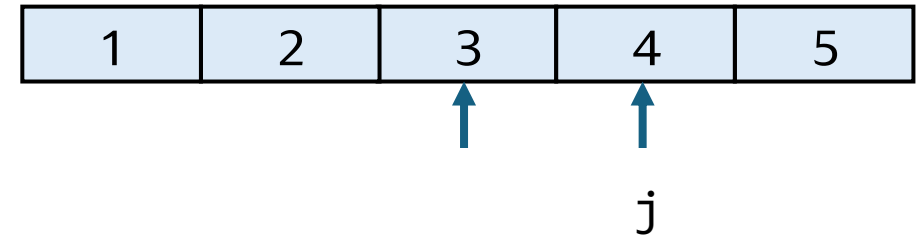
```
void bubble_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



`i = 2; j = 2; swapped = false`

# Bubble Sort with Early Termination

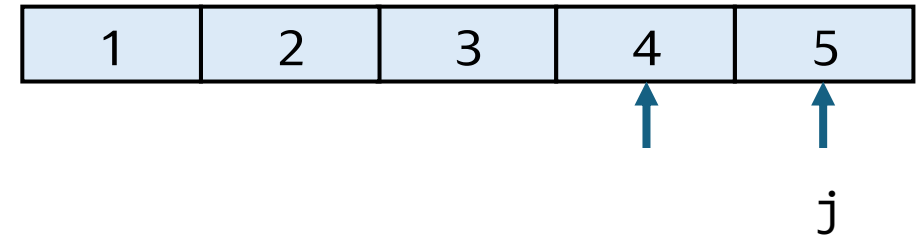
```
void bubble_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



`i = 2; j = 3; swapped = false`

# Bubble Sort with Early Termination

```
void bubble_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```

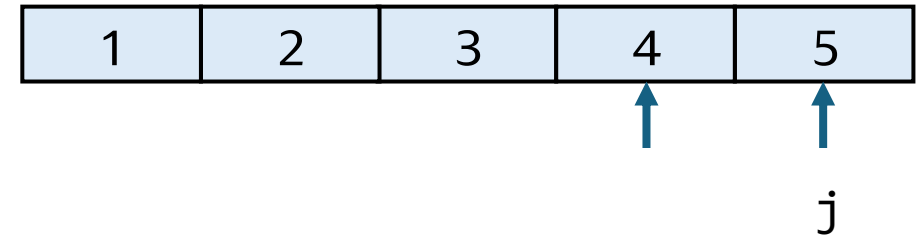


`i = 2; j = 4; swapped = false`

# Bubble Sort with Early Termination

```
void bubble_sort_v1(int *arr, size_t sz) {
    if (sz <= 1) return;

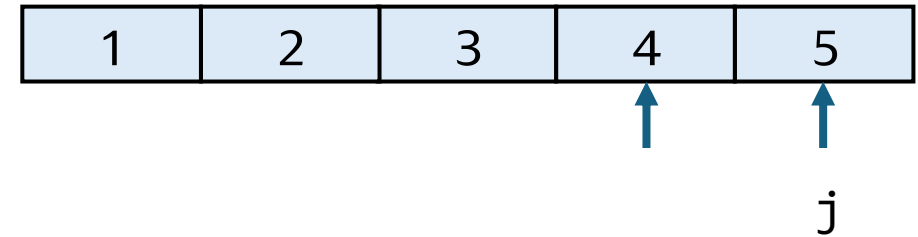
    for (size_t i = 0; i < sz - 1; i++) {
        bool swapped = false;
        for (size_t j = 1; j < sz; j++) {
            if (arr[j] < arr[j - 1]) {
                swap_int(&arr[j - 1], &arr[j]);
                swapped = true;
            }
        }
        if (!swapped) break;
    }
}
```



`i = 2; j = 4; swapped = false`

# Bubble Sort with Early Termination

```
void bubble_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



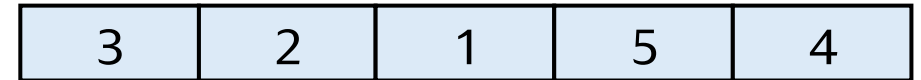
Worst-case:  $(n - 1)^2$  comparisons +  $n(n - 1)/2$  swaps  
Best-case:  $(n - 1)$  comparisons

# Bubble Sort with Early Termination + Sorted Region

```
void bubble_sort_v2(int *arr, size_t sz) {
    if (sz <= 1) return;

    for (size_t i = 0; i < sz - 1; i++) {
        bool swapped = false;
        for (size_t j = 1; j < sz - i; j++) {
            if (arr[j] < arr[j - 1]) {
                swap_int(&arr[j - 1], &arr[j]);
                swapped = true;
            }
        }

        if (!swapped) break;
    }
}
```

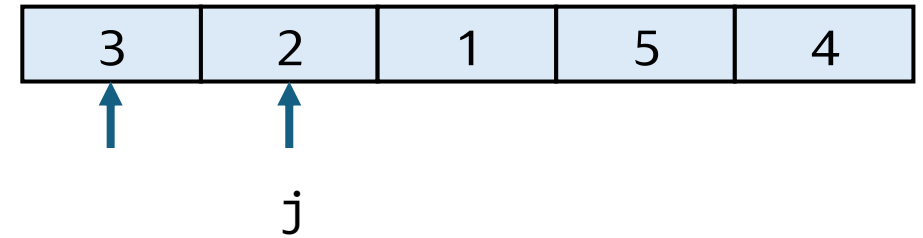


# Bubble Sort with Early Termination + Sorted Region

```
void bubble_sort_v2(int *arr, size_t sz) {  
    if (sz <= 1) return;
```

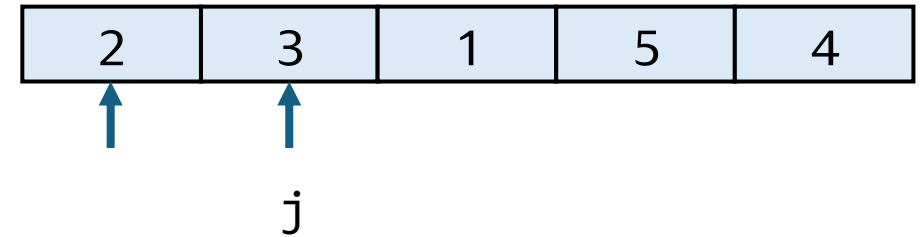
```
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz - i; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
    }
```

```
    if (!swapped) break;    i = 0; j = 1; swapped = false  
}
```



# Bubble Sort with Early Termination + Sorted Region

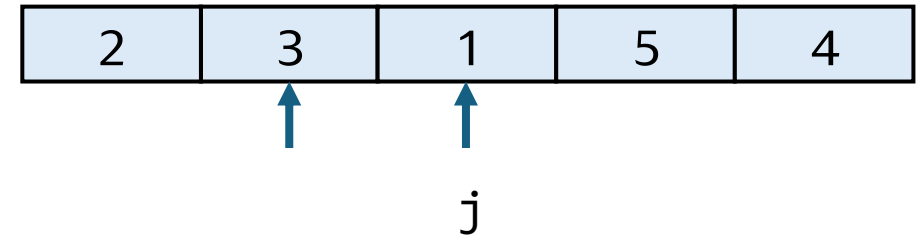
```
void bubble_sort_v2(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz - i; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



`i = 0; j = 1; swapped = true`

# Bubble Sort with Early Termination + Sorted Region

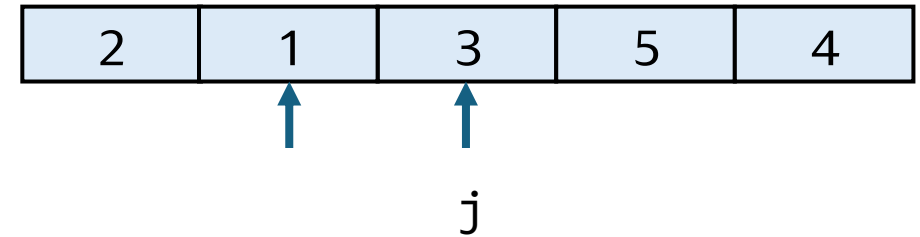
```
void bubble_sort_v2(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz - i; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
        if (!swapped) break;  
    }  
}
```



`i = 0; j = 2; swapped = true`

# Bubble Sort with Early Termination + Sorted Region

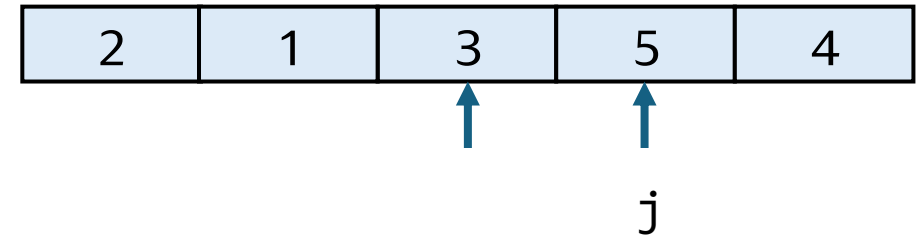
```
void bubble_sort_v2(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz - i; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



`i = 0; j = 2; swapped = true`

# Bubble Sort with Early Termination + Sorted Region

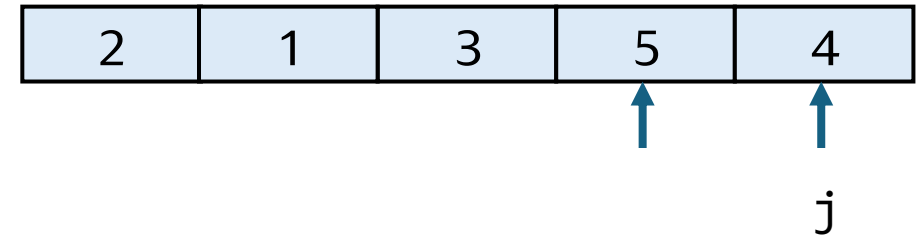
```
void bubble_sort_v2(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz - i; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



$i = 0; j = 3; \text{swapped} = \text{true}$

# Bubble Sort with Early Termination + Sorted Region

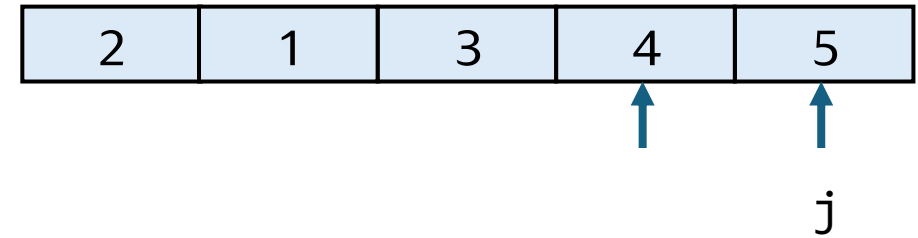
```
void bubble_sort_v2(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz - i; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



`i = 0; j = 4; swapped = true`

# Bubble Sort with Early Termination + Sorted Region

```
void bubble_sort_v2(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz - i; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



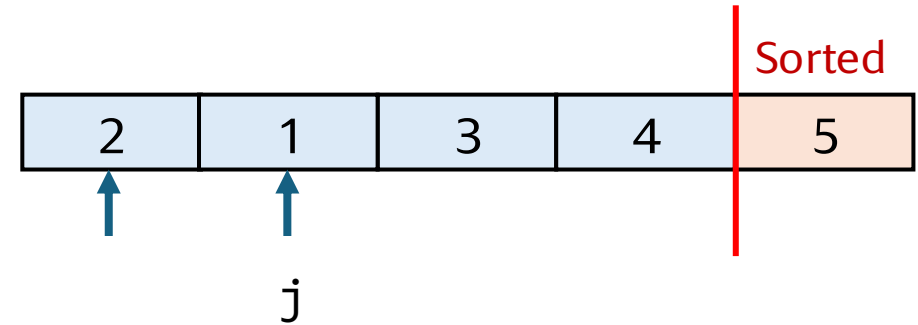
`i = 0; j = 4; swapped = true`

# Bubble Sort with Early Termination + Sorted Region

```
void bubble_sort_v2(int *arr, size_t sz) {  
    if (sz <= 1) return;
```

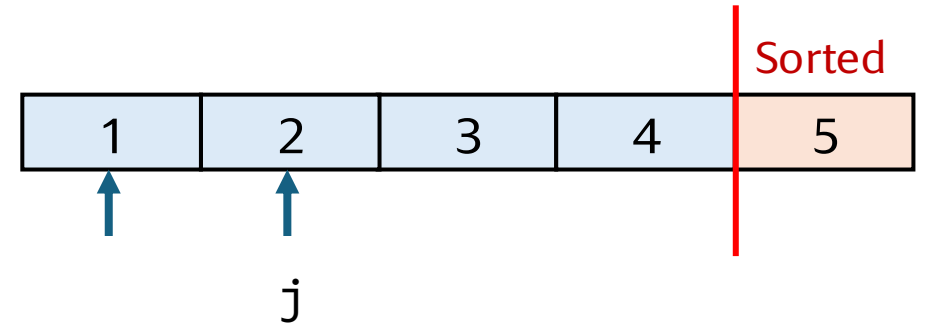
```
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz - i; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
    }
```

```
    if (!swapped) break;    i = 1; j = 1; swapped = false  
}
```



# Bubble Sort with Early Termination + Sorted Region

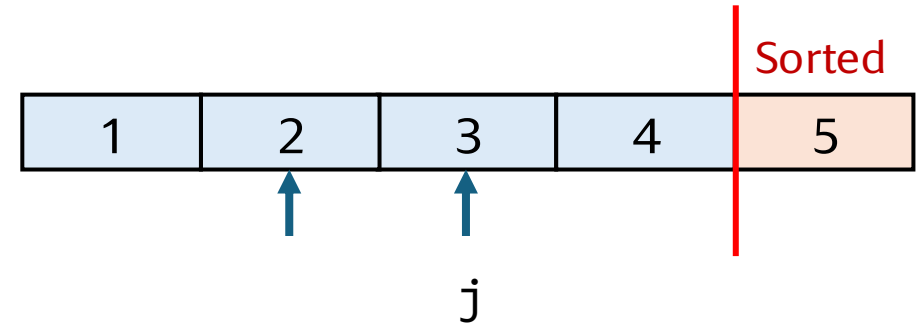
```
void bubble_sort_v2(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz - i; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



`i = 1; j = 1; swapped = true`

# Bubble Sort with Early Termination + Sorted Region

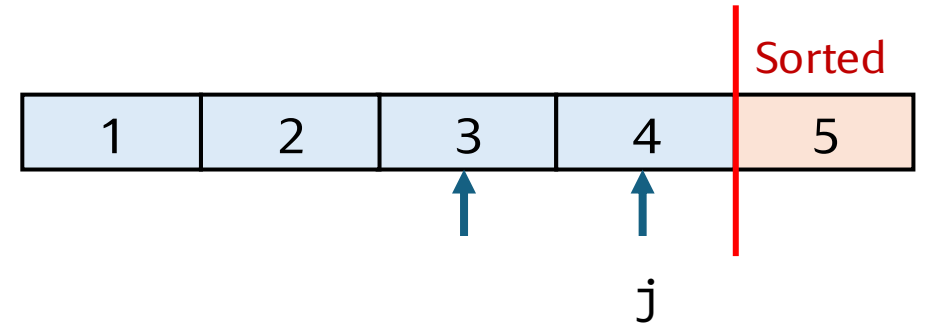
```
void bubble_sort_v2(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz - i; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



$i = 1; j = 2; \text{swapped} = \text{true}$

# Bubble Sort with Early Termination + Sorted Region

```
void bubble_sort_v2(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz - i; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



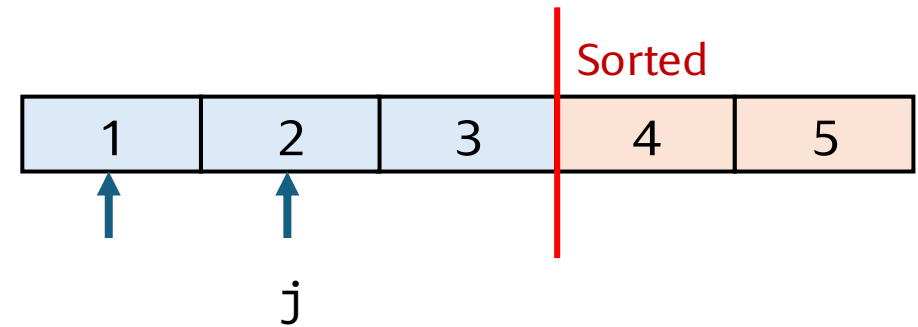
$i = 1; j = 3; \text{swapped} = \text{true}$

# Bubble Sort with Early Termination + Sorted Region

```
void bubble_sort_v2(int *arr, size_t sz) {  
    if (sz <= 1) return;
```

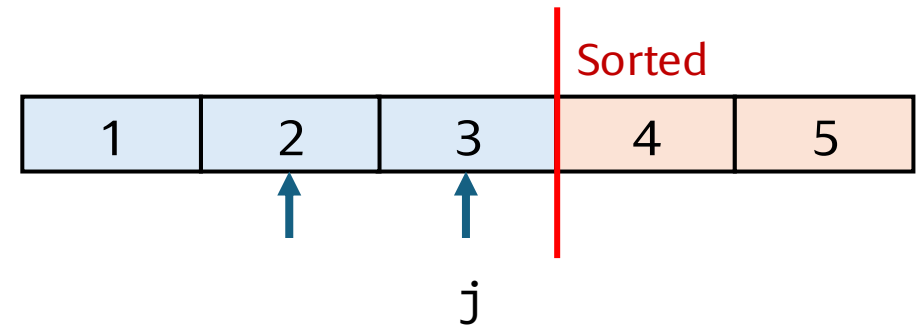
```
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz - i; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
    }
```

```
    if (!swapped) break;    i = 2; j = 1; swapped = false  
}
```



# Bubble Sort with Early Termination + Sorted Region

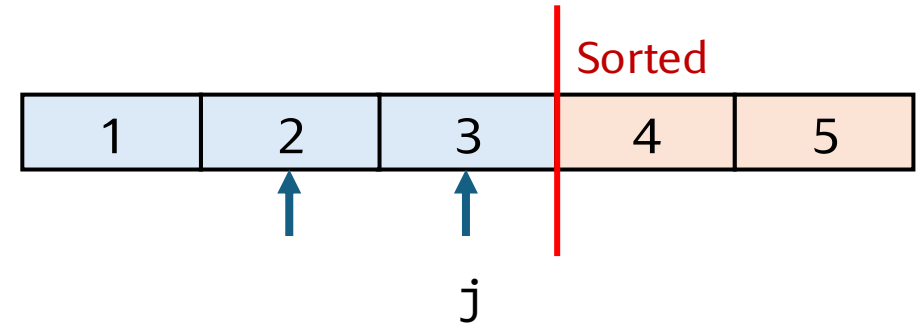
```
void bubble_sort_v2(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz - i; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
  
        if (!swapped) break;  
    }  
}
```



$i = 2; j = 2; \text{swapped} = \text{false}$

# Bubble Sort with Early Termination + Sorted Region

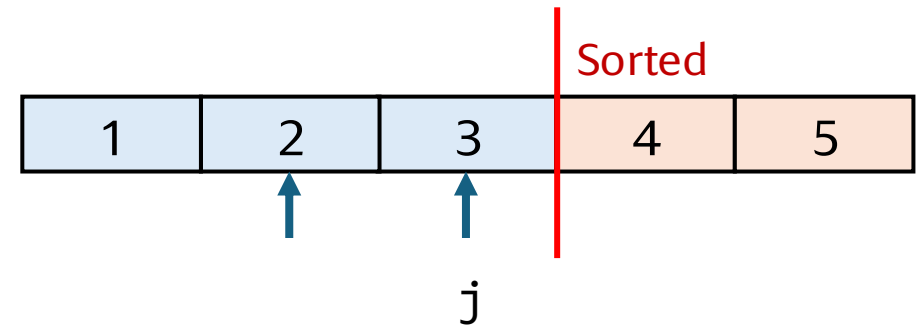
```
void bubble_sort_v2(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz - i; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
        if (!swapped) break;  
    }  
}
```



$i = 2; j = 2; \text{swapped} = \text{false}$

# Bubble Sort with Early Termination + Sorted Region

```
void bubble_sort_v2(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t i = 0; i < sz - 1; i++) {  
        bool swapped = false;  
        for (size_t j = 1; j < sz - i; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                swapped = true;  
            }  
        }  
        if (!swapped) break;  
    }  
}
```

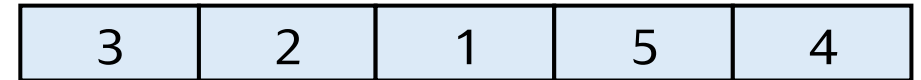


Worst-case:  $n(n - 1)/2$  (comparisons + swaps)

Best-case:  $(n - 1)$  comparisons

# Bubble Sort with Early Termination + Better Sorted Region

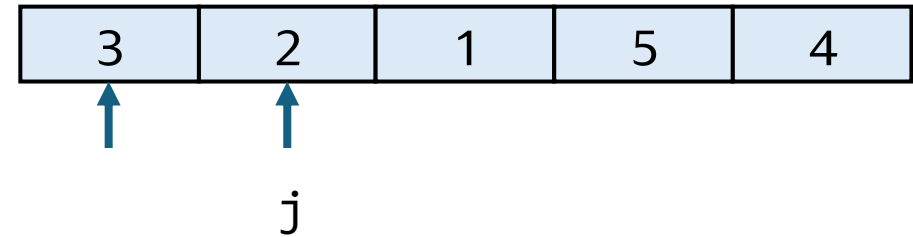
```
void bubble_sort_v3(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    size_t end = sz;  
    for (size_t i = 0; i < sz - 1; i++) {  
        size_t last_swapped = 0;  
        for (size_t j = 1; j < end; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                last_swapped = j;  
            }  
        }  
        if (!last_swapped) break;  
        end = last_swapped;  
    }  
}
```



# Bubble Sort with Early Termination + Better Sorted Region

```
void bubble_sort_v3(int *arr, size_t sz) {  
    if (sz <= 1) return;
```

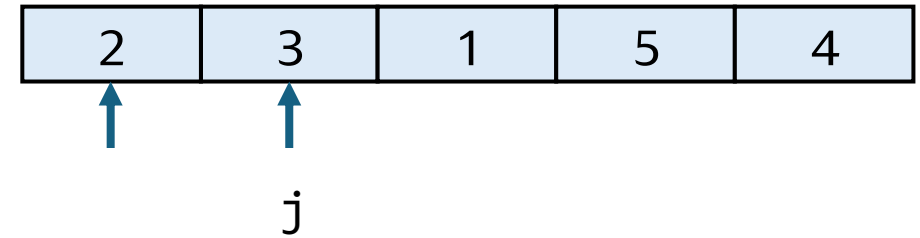
```
    size_t end = sz;  
    for (size_t i = 0; i < sz - 1; i++) {  
        size_t last_swapped = 0;  
        for (size_t j = 1; j < end; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                last_swapped = j;  
            }  
        }  
        if (!last_swapped) break;  
        end = last_swapped;  
    }  
}
```



`end = 5; i = 0; j = 1; last_swapped = 0`

# Bubble Sort with Early Termination + Better Sorted Region

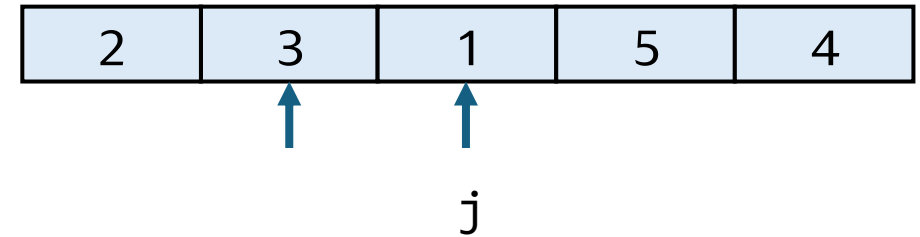
```
void bubble_sort_v3(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    size_t end = sz;  
    for (size_t i = 0; i < sz - 1; i++) {  
        size_t last_swapped = 0;  
        for (size_t j = 1; j < end; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                last_swapped = j;  
            }  
        }  
        if (!last_swapped) break;  
        end = last_swapped;  
    }  
}
```



`end = 5; i = 0; j = 1; last_swapped = 1`

# Bubble Sort with Early Termination + Better Sorted Region

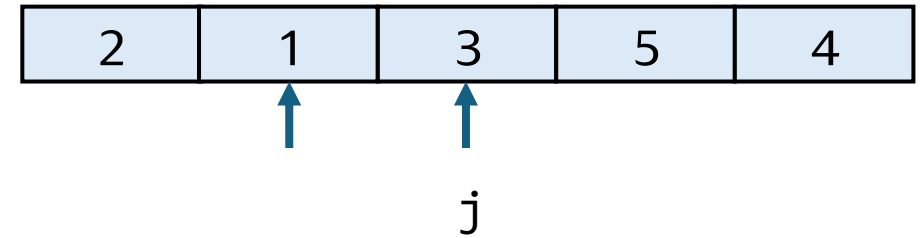
```
void bubble_sort_v3(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    size_t end = sz;  
    for (size_t i = 0; i < sz - 1; i++) {  
        size_t last_swapped = 0;  
        for (size_t j = 1; j < end; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                last_swapped = j;  
            }  
        }  
        if (!last_swapped) break;  
        end = last_swapped;  
    }  
}
```



`end = 5; i = 0; j = 2; last_swapped = 1`

# Bubble Sort with Early Termination + Better Sorted Region

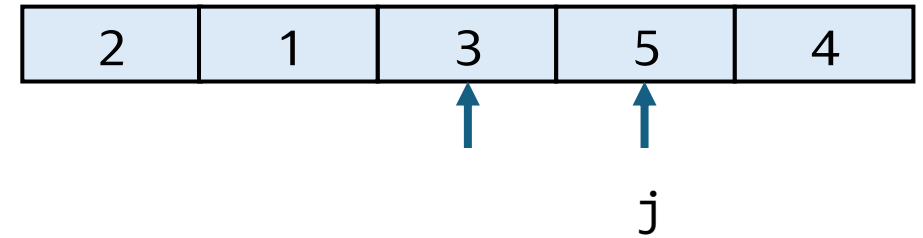
```
void bubble_sort_v3(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    size_t end = sz;  
    for (size_t i = 0; i < sz - 1; i++) {  
        size_t last_swapped = 0;  
        for (size_t j = 1; j < end; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                last_swapped = j;  
            }  
        }  
        if (!last_swapped) break;  
        end = last_swapped;  
    }  
}
```



`end = 5; i = 0; j = 2; last_swapped = 2`

# Bubble Sort with Early Termination + Better Sorted Region

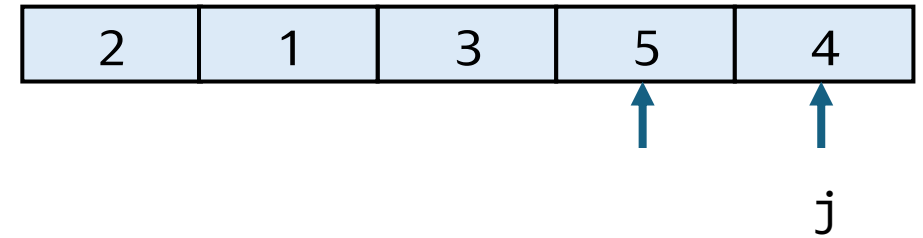
```
void bubble_sort_v3(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    size_t end = sz;  
    for (size_t i = 0; i < sz - 1; i++) {  
        size_t last_swapped = 0;  
        for (size_t j = 1; j < end; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                last_swapped = j;  
            }  
        }  
        if (!last_swapped) break;  
        end = last_swapped;  
    }  
}
```



`end = 5; i = 0; j = 3; last_swapped = 2`

# Bubble Sort with Early Termination + Better Sorted Region

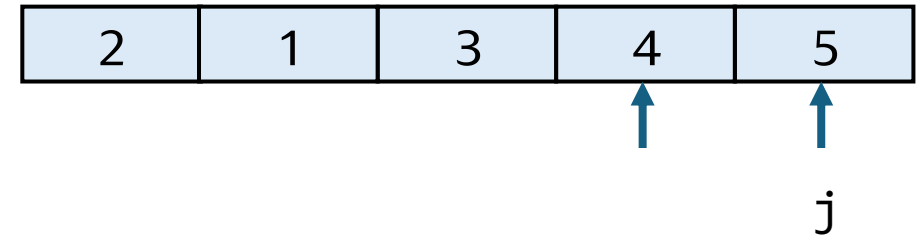
```
void bubble_sort_v3(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    size_t end = sz;  
    for (size_t i = 0; i < sz - 1; i++) {  
        size_t last_swapped = 0;  
        for (size_t j = 1; j < end; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                last_swapped = j;  
            }  
        }  
        if (!last_swapped) break;  
        end = last_swapped;  
    }  
}
```



`end = 5; i = 0; j = 4; last_swapped = 2`

# Bubble Sort with Early Termination + Better Sorted Region

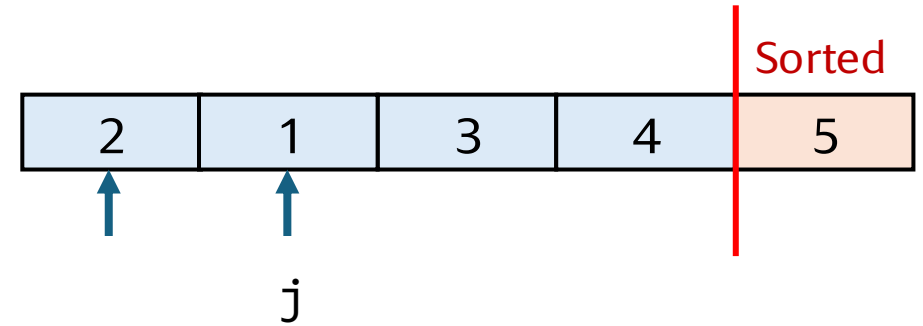
```
void bubble_sort_v3(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    size_t end = sz;  
    for (size_t i = 0; i < sz - 1; i++) {  
        size_t last_swapped = 0;  
        for (size_t j = 1; j < end; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                last_swapped = j;  
            }  
        }  
        if (!last_swapped) break;  
        end = last_swapped;  
    }  
}
```



`end = 5; i = 0; j = 4; last_swapped = 4`

# Bubble Sort with Early Termination + Better Sorted Region

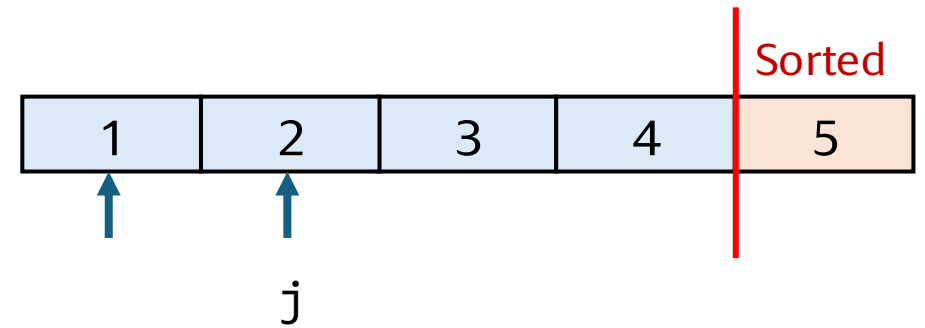
```
void bubble_sort_v3(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    size_t end = sz;  
    for (size_t i = 0; i < sz - 1; i++) {  
        size_t last_swapped = 0;  
        for (size_t j = 1; j < end; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                last_swapped = j;  
            }  
        }  
        if (!last_swapped) break;  
        end = last_swapped;  
    }  
}
```



`end = 4; i = 0; j = 1; last_swapped = 0`

# Bubble Sort with Early Termination + Better Sorted Region

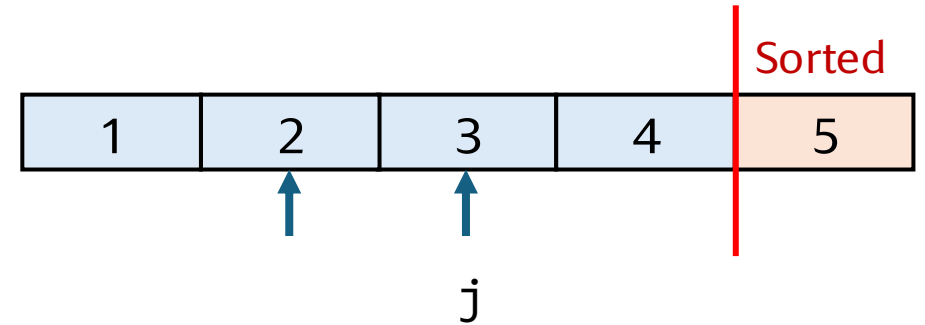
```
void bubble_sort_v3(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    size_t end = sz;  
    for (size_t i = 0; i < sz - 1; i++) {  
        size_t last_swapped = 0;  
        for (size_t j = 1; j < end; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                last_swapped = j;  
            }  
        }  
        if (!last_swapped) break;  
        end = last_swapped;  
    }  
}
```



`end = 4; i = 1; j = 1; last_swapped = 1`

# Bubble Sort with Early Termination + Better Sorted Region

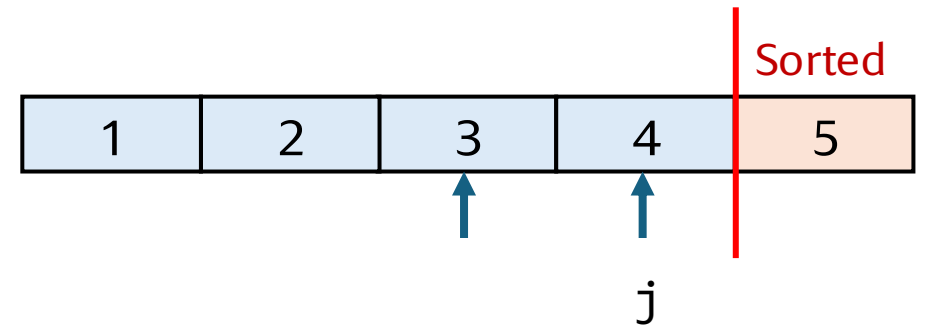
```
void bubble_sort_v3(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    size_t end = sz;  
    for (size_t i = 0; i < sz - 1; i++) {  
        size_t last_swapped = 0;  
        for (size_t j = 1; j < end; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                last_swapped = j;  
            }  
        }  
        if (!last_swapped) break;  
        end = last_swapped;  
    }  
}
```



`end = 4; i = 1; j = 2; last_swapped = 1`

# Bubble Sort with Early Termination + Better Sorted Region

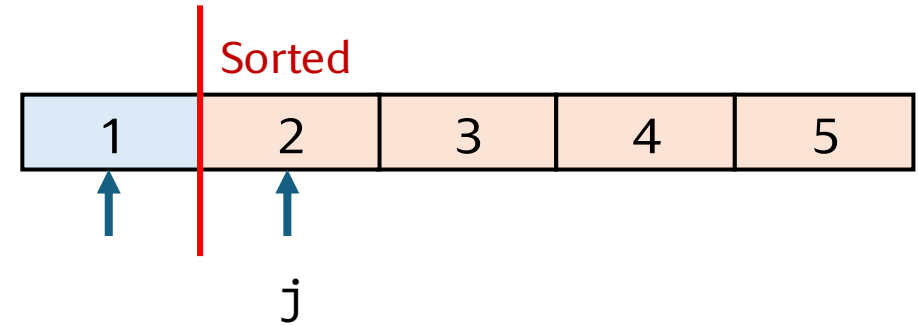
```
void bubble_sort_v3(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    size_t end = sz;  
    for (size_t i = 0; i < sz - 1; i++) {  
        size_t last_swapped = 0;  
        for (size_t j = 1; j < end; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                last_swapped = j;  
            }  
        }  
        if (!last_swapped) break;  
        end = last_swapped;  
    }  
}
```



`end = 4; i = 1; j = 3; last_swapped = 1`

# Bubble Sort with Early Termination + Better Sorted Region

```
void bubble_sort_v3(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    size_t end = sz;  
    for (size_t i = 0; i < sz - 1; i++) {  
        size_t last_swapped = 0;  
        for (size_t j = 1; j < end; j++) {  
            if (arr[j] < arr[j - 1]) {  
                swap_int(&arr[j - 1], &arr[j]);  
                last_swapped = j;  
            }  
        }  
        if (!last_swapped) break;  
        end = last_swapped;  
    }  
}
```

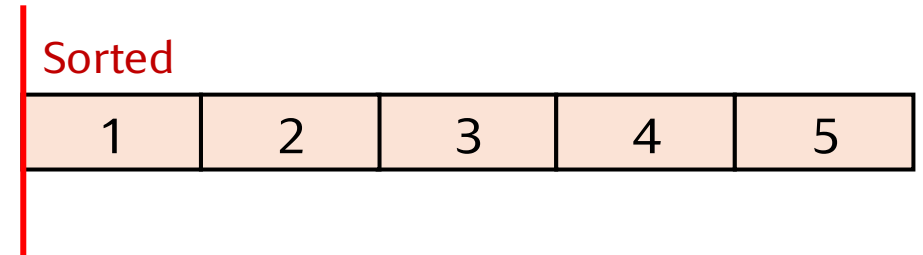


`end = 1; i = 2; j = 1; last_swapped = 0`

# Bubble Sort with Early Termination + Better Sorted Region

```
void bubble_sort_v3(int *arr, size_t sz) {
    if (sz <= 1) return;

    size_t end = sz;
    for (size_t i = 0; i < sz - 1; i++) {
        size_t last_swapped = 0;
        for (size_t j = 1; j < end; j++) {
            if (arr[j] < arr[j - 1]) {
                swap_int(&arr[j - 1], &arr[j]);
                last_swapped = j;
            }
        }
        if (!last_swapped) break;
        end = last_swapped;
    }
}
```



Worst-case:  $n(n - 1)/2$  (comparisons + swaps)

Best-case:  $(n - 1)$  comparisons

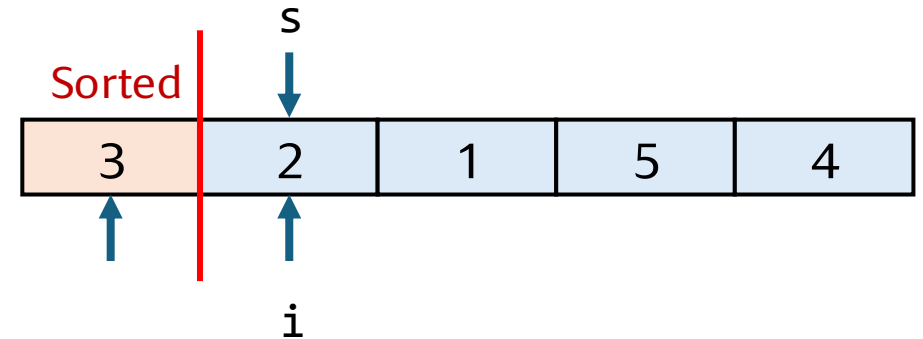
# Naïve Insertion Sort

```
void insertion_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        for (size_t i = s; i > 0; i--) {  
            if (arr[i] < arr[i - 1])  
                swap_int(&arr[i], &arr[i - 1]);  
            else  
                break;  
        }  
    }  
}
```



# Naïve Insertion Sort

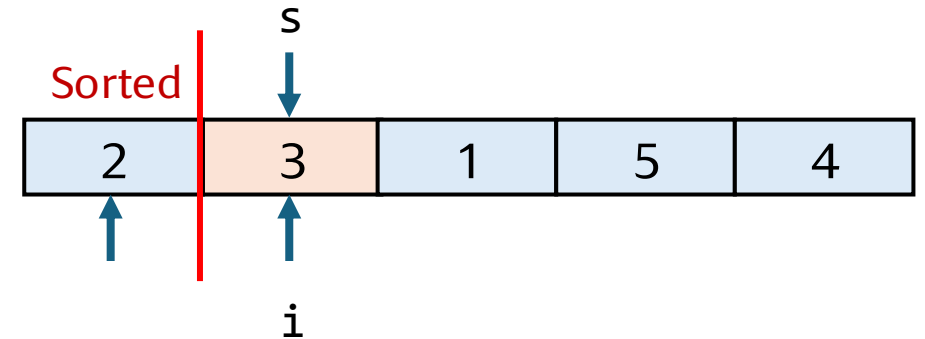
```
void insertion_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        for (size_t i = s; i > 0; i--) {  
            if (arr[i] < arr[i - 1])  
                swap_int(&arr[i], &arr[i - 1]);  
            else  
                break;  
        }  
    }  
}
```



$s = 1; i = 1$

# Naïve Insertion Sort

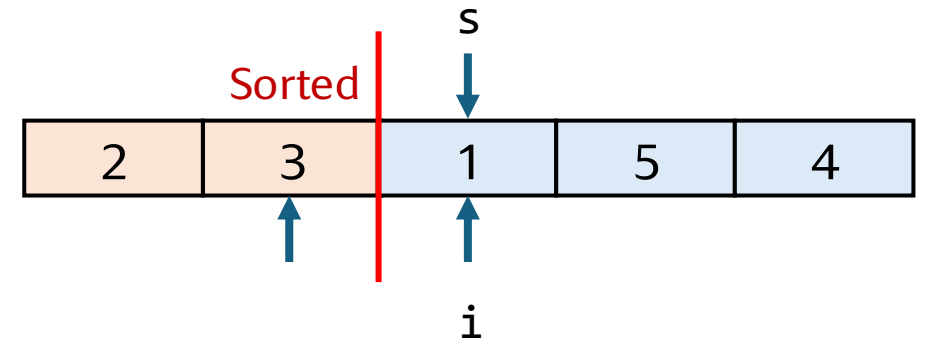
```
void insertion_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        for (size_t i = s; i > 0; i--) {  
            if (arr[i] < arr[i - 1])  
                swap_int(&arr[i], &arr[i - 1]);  
            else  
                break;  
        }  
    }  
}
```



$s = 1; i = 1$

# Naïve Insertion Sort

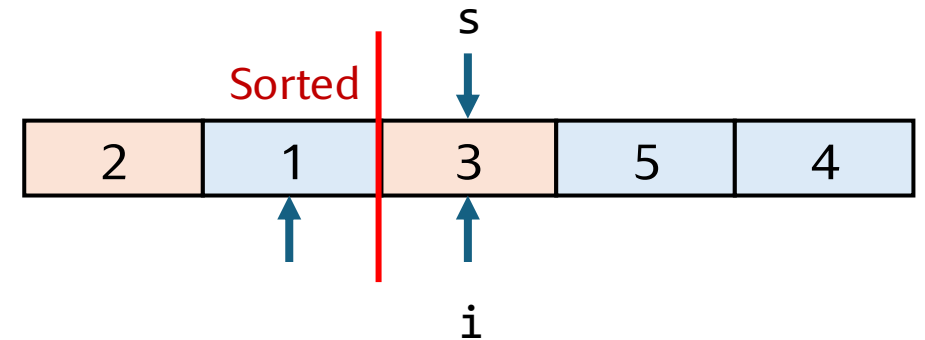
```
void insertion_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        for (size_t i = s; i > 0; i--) {  
            if (arr[i] < arr[i - 1])  
                swap_int(&arr[i], &arr[i - 1]);  
            else  
                break;  
        }  
    }  
}
```



$s = 2; i = 2$

# Naïve Insertion Sort

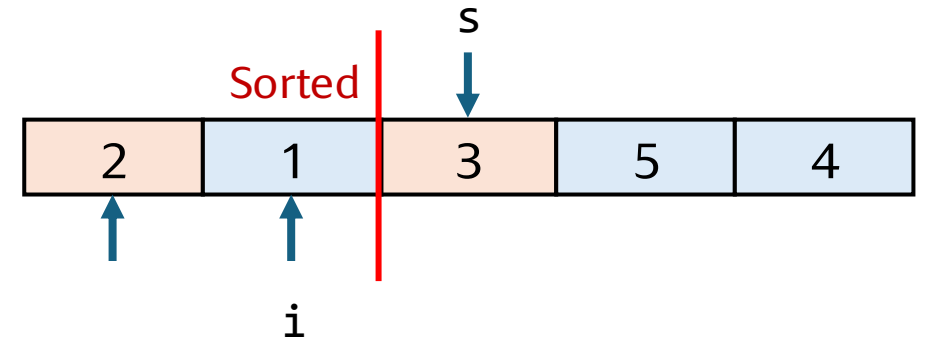
```
void insertion_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        for (size_t i = s; i > 0; i--) {  
            if (arr[i] < arr[i - 1])  
                swap_int(&arr[i], &arr[i - 1]);  
            else  
                break;  
        }  
    }  
}
```



$s = 2; i = 2$

# Naïve Insertion Sort

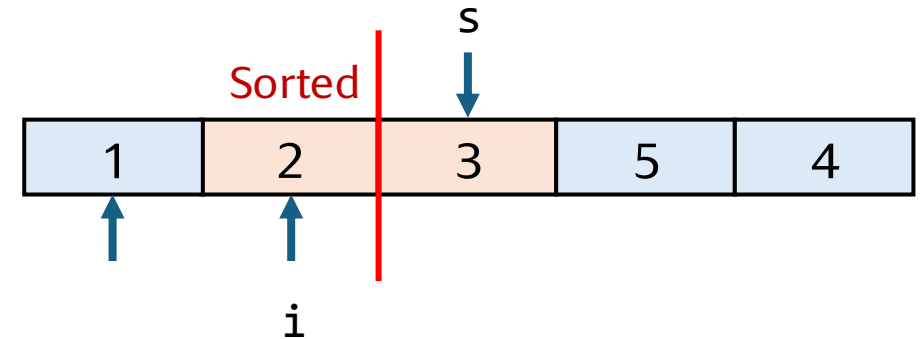
```
void insertion_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        for (size_t i = s; i > 0; i--) {  
            if (arr[i] < arr[i - 1])  
                swap_int(&arr[i], &arr[i - 1]);  
            else  
                break;  
        }  
    }  
}
```



$s = 2; i = 1$

# Naïve Insertion Sort

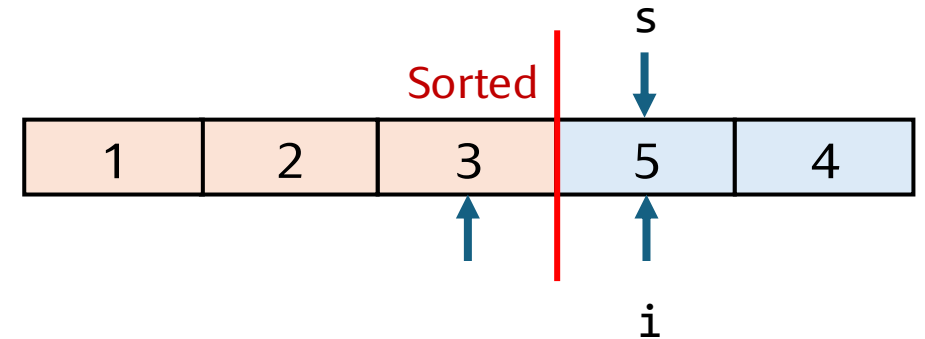
```
void insertion_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        for (size_t i = s; i > 0; i--) {  
            if (arr[i] < arr[i - 1])  
                swap_int(&arr[i], &arr[i - 1]);  
            else  
                break;  
        }  
    }  
}
```



$s = 2; i = 1$

# Naïve Insertion Sort

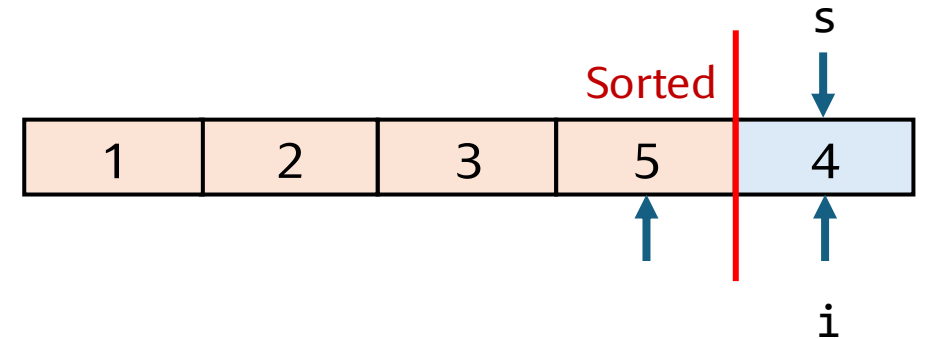
```
void insertion_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        for (size_t i = s; i > 0; i--) {  
            if (arr[i] < arr[i - 1])  
                swap_int(&arr[i], &arr[i - 1]);  
            else  
                break;  
        }  
    }  
}
```



$s = 3; i = 3$

# Naïve Insertion Sort

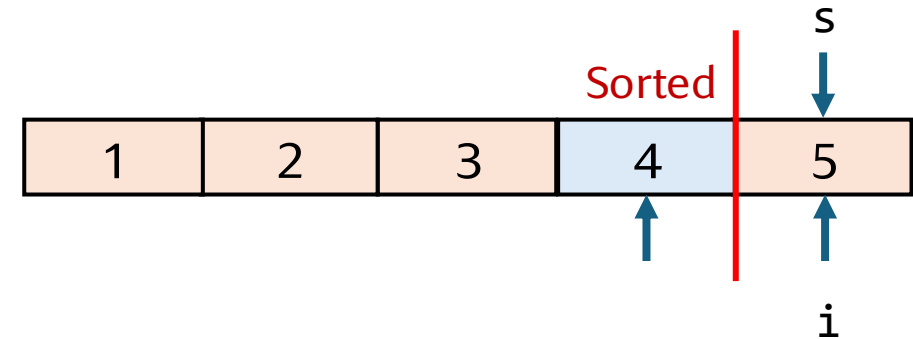
```
void insertion_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        for (size_t i = s; i > 0; i--) {  
            if (arr[i] < arr[i - 1])  
                swap_int(&arr[i], &arr[i - 1]);  
            else  
                break;  
        }  
    }  
}
```



$s = 4; i = 4$

# Naïve Insertion Sort

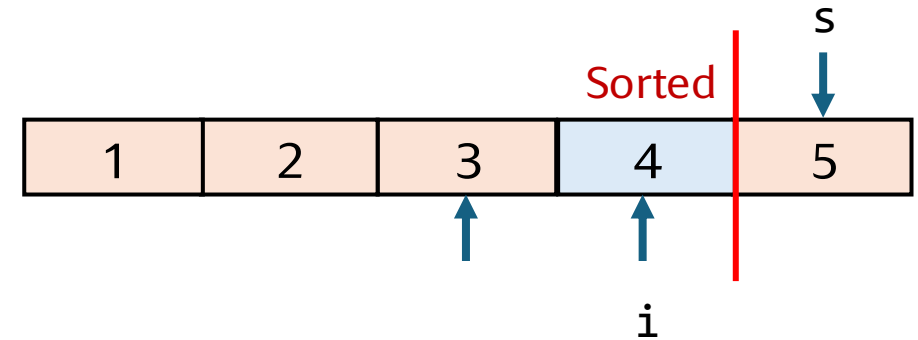
```
void insertion_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        for (size_t i = s; i > 0; i--) {  
            if (arr[i] < arr[i - 1])  
                swap_int(&arr[i], &arr[i - 1]);  
            else  
                break;  
        }  
    }  
}
```



$s = 4; i = 4$

# Naïve Insertion Sort

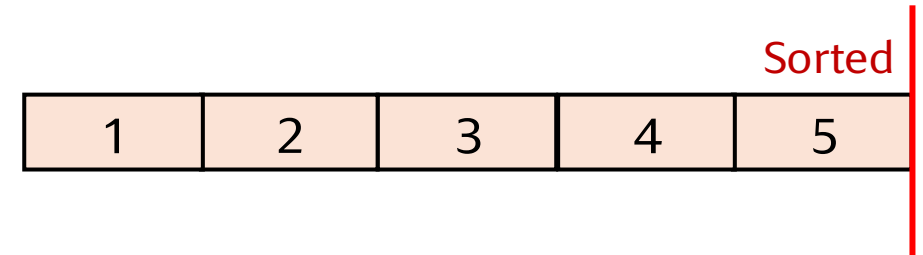
```
void insertion_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        for (size_t i = s; i > 0; i--) {  
            if (arr[i] < arr[i - 1])  
                swap_int(&arr[i], &arr[i - 1]);  
            else  
                break;  
        }  
    }  
}
```



$s = 4; i = 3$

# Naïve Insertion Sort

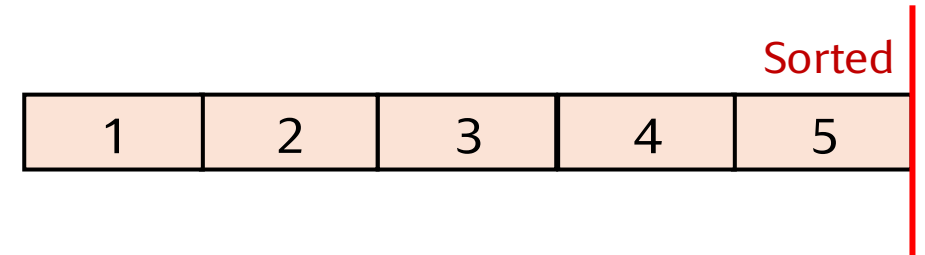
```
void insertion_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        for (size_t i = s; i > 0; i--) {  
            if (arr[i] < arr[i - 1])  
                swap_int(&arr[i], &arr[i - 1]);  
            else  
                break;  
        }  
    }  
}
```



$s = 4; i = 3$

# Naïve Insertion Sort

```
void insertion_sort_v0(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        for (size_t i = s; i > 0; i--) {  
            if (arr[i] < arr[i - 1])  
                swap_int(&arr[i], &arr[i - 1]);  
            else  
                break;  
        }  
    }  
}
```

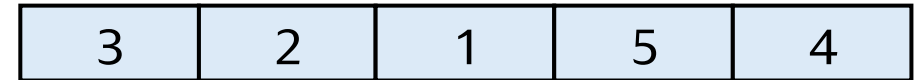


Worst-case:  $n(n - 1)/2$  (comparisons + swaps)

Best-case:  $(n - 1)$  comparisons

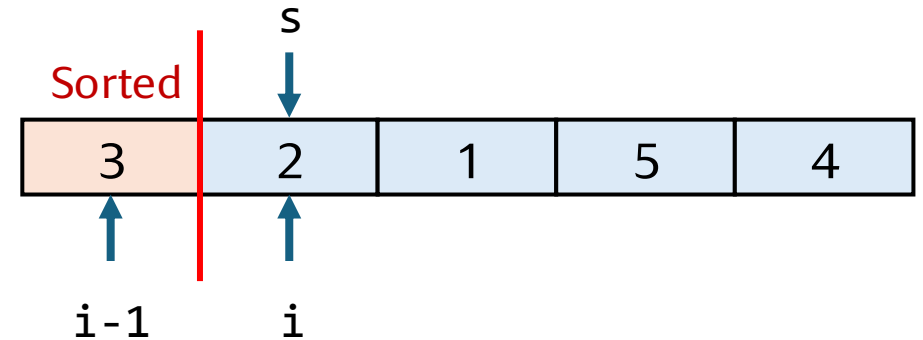
# Insertion Sort with Moving

```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



# Insertion Sort with Moving

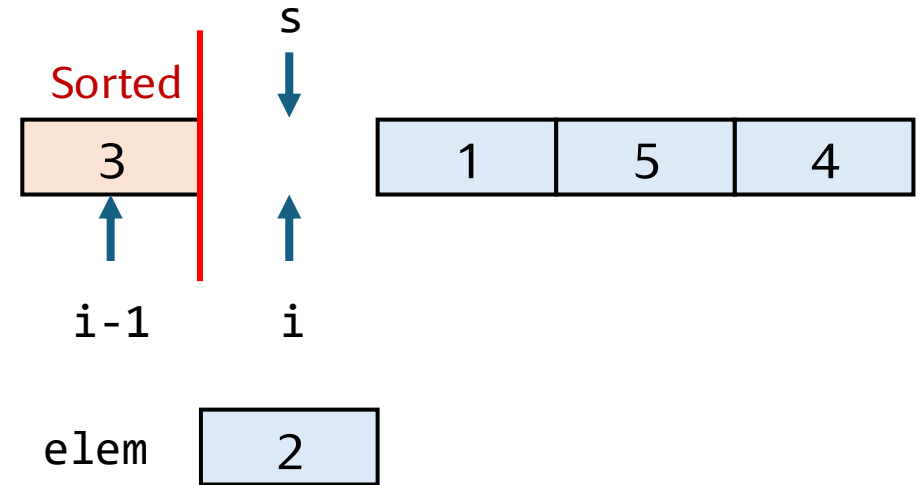
```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



$s = 1; i = 1$

# Insertion Sort with Moving

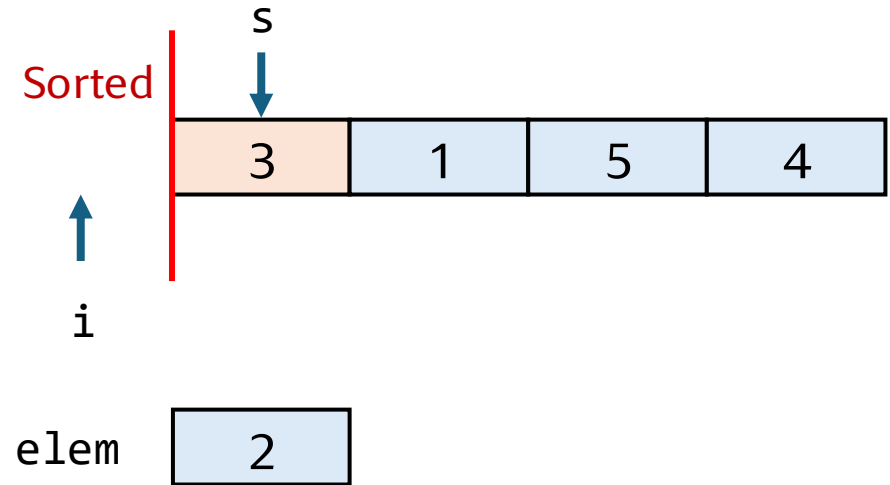
```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



`s = 1; i = 1; elem = 2`

# Insertion Sort with Moving

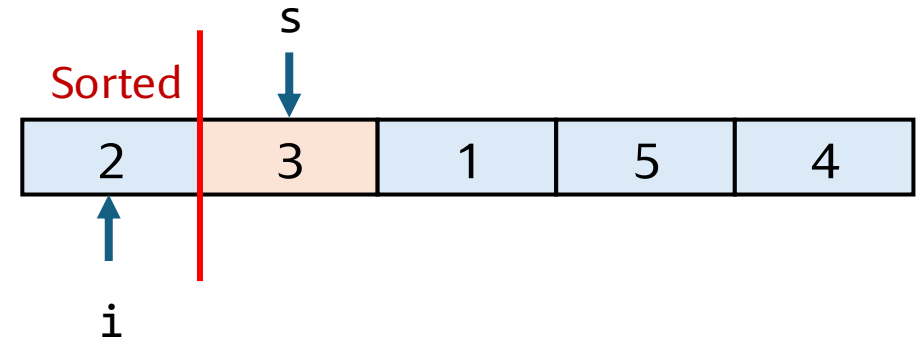
```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



`s = 1; i = 0; elem = 2`

# Insertion Sort with Moving

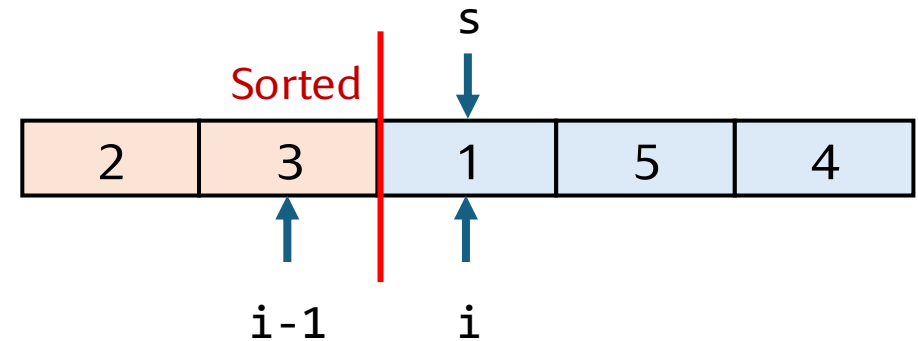
```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



`s = 1; i = 0; elem = 2`

# Insertion Sort with Moving

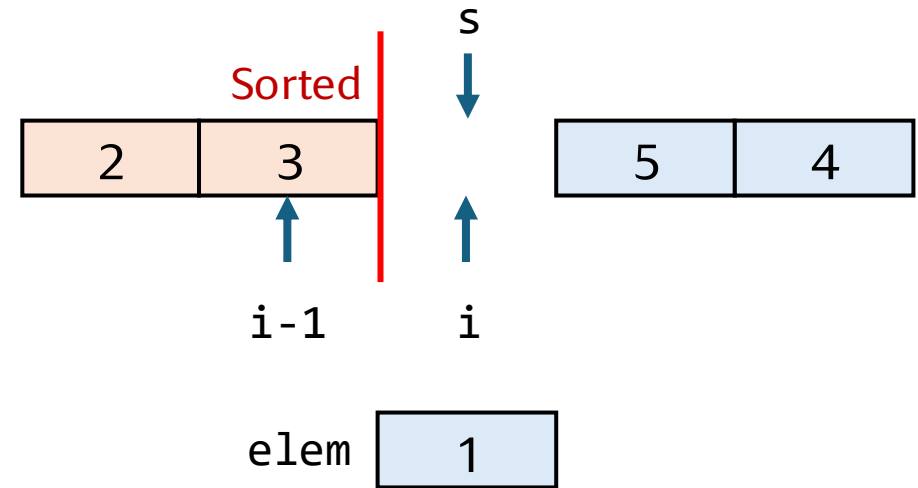
```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



$s = 2; i = 2$

# Insertion Sort with Moving

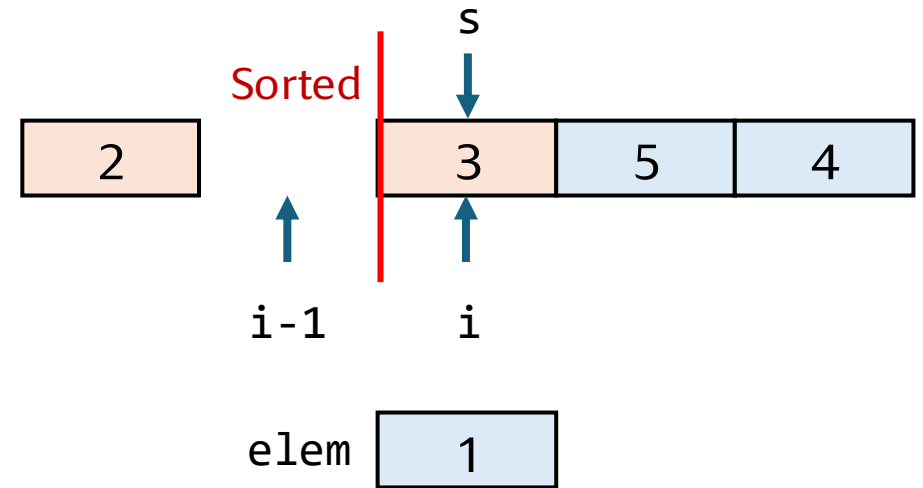
```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



$s = 2; i = 2; elem = 1$

# Insertion Sort with Moving

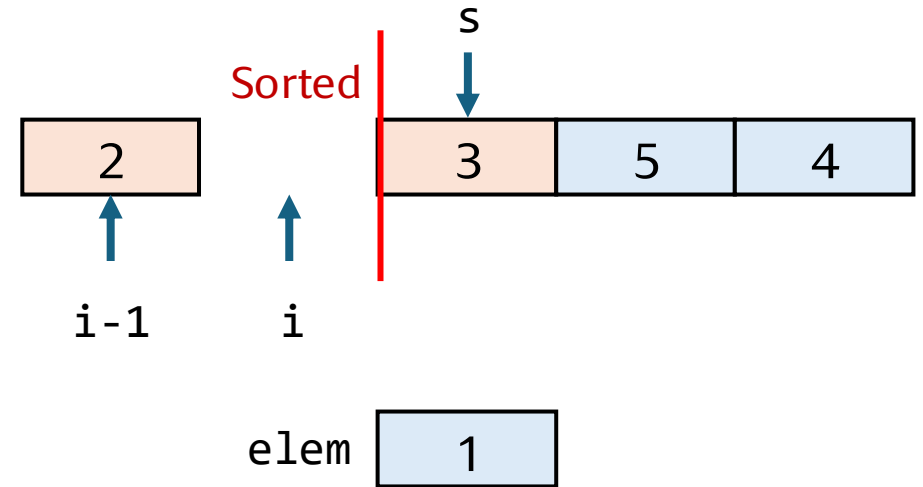
```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



$s = 2; i = 2; elem = 1$

# Insertion Sort with Moving

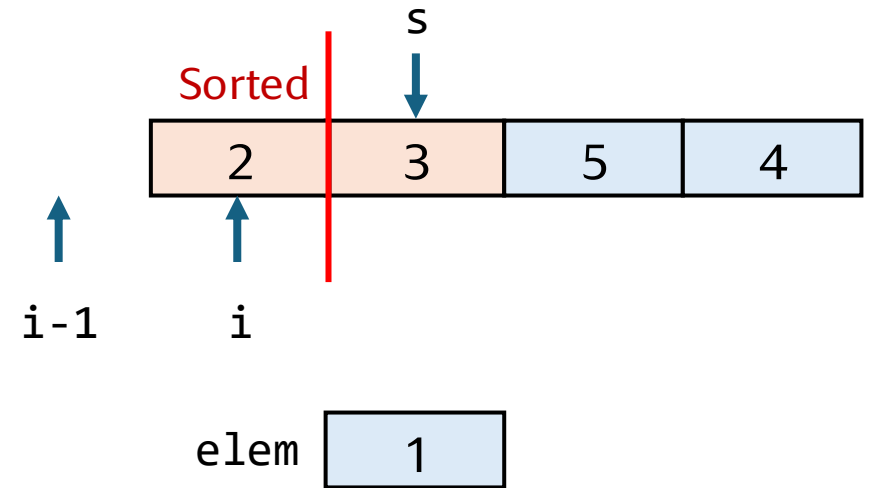
```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



`s = 2; i = 1; elem = 1`

# Insertion Sort with Moving

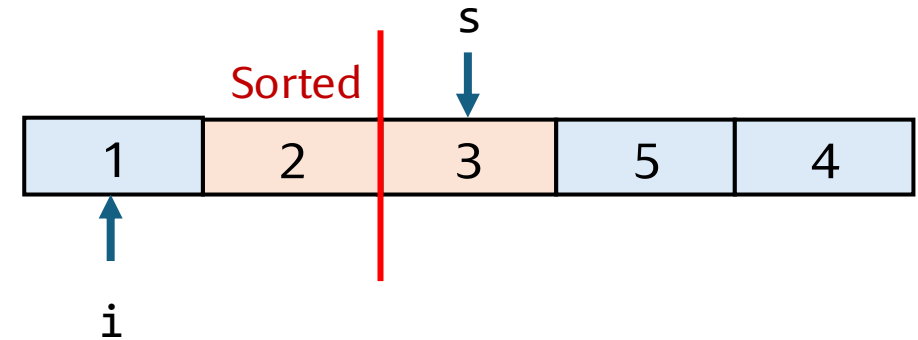
```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



`s = 2; i = 1; elem = 1`

# Insertion Sort with Moving

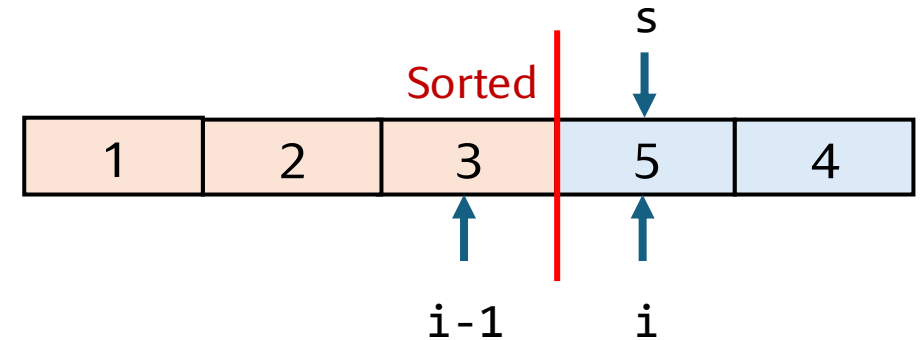
```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



`s = 2; i = 0; elem = 1`

# Insertion Sort with Moving

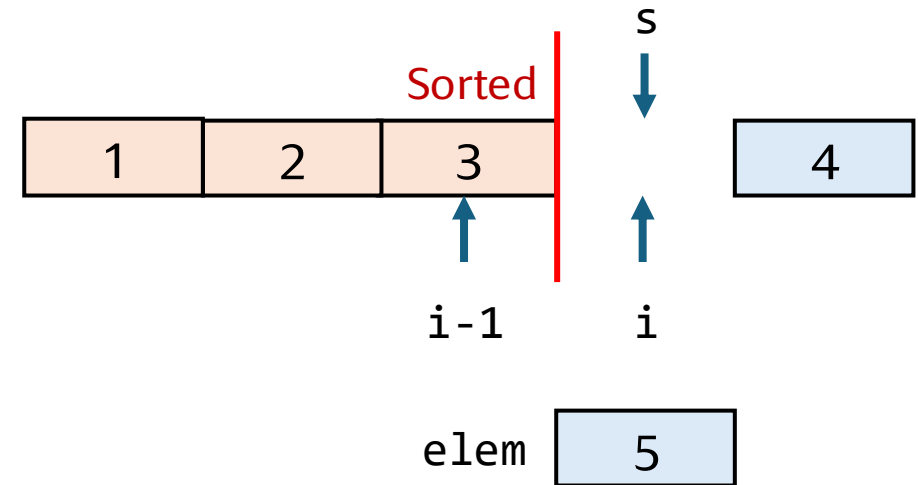
```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



$s = 3; i = 3$

# Insertion Sort with Moving

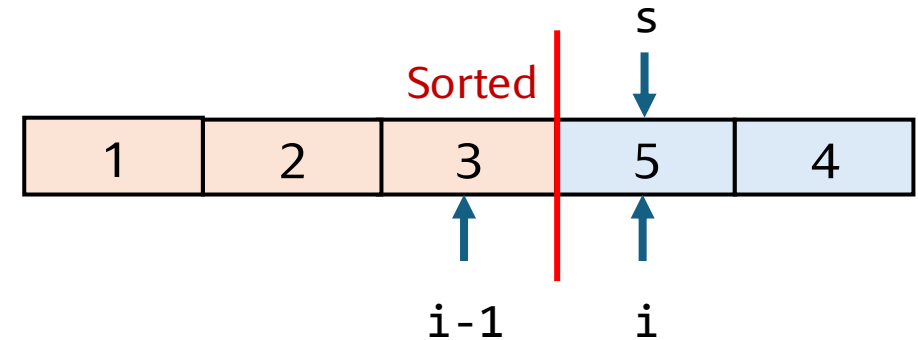
```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



`s = 3; i = 3; elem = 5`

# Insertion Sort with Moving

```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



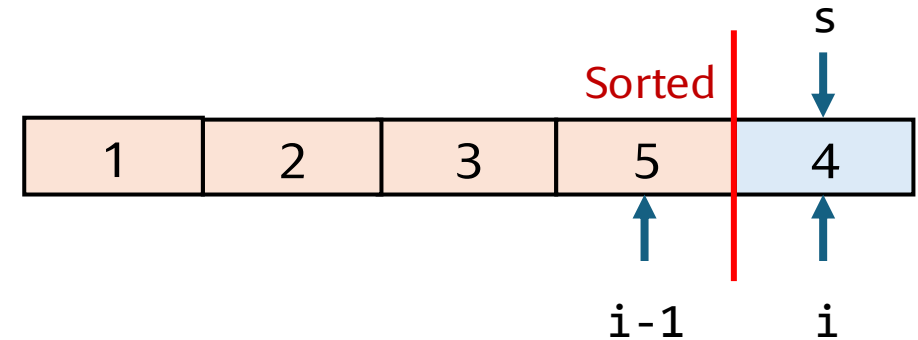
`s = 3; i = 3; elem = 5`

# Insertion Sort with Moving

```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;
```

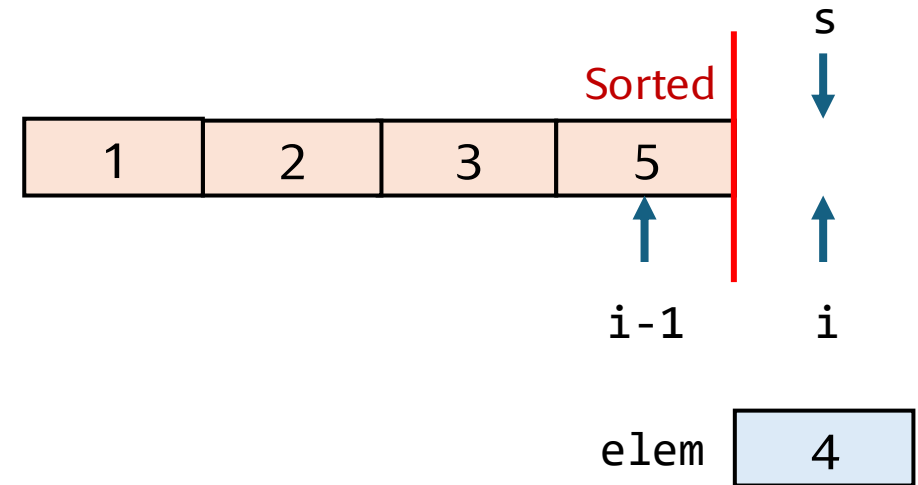
```
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```

$s = 4; i = 4$



# Insertion Sort with Moving

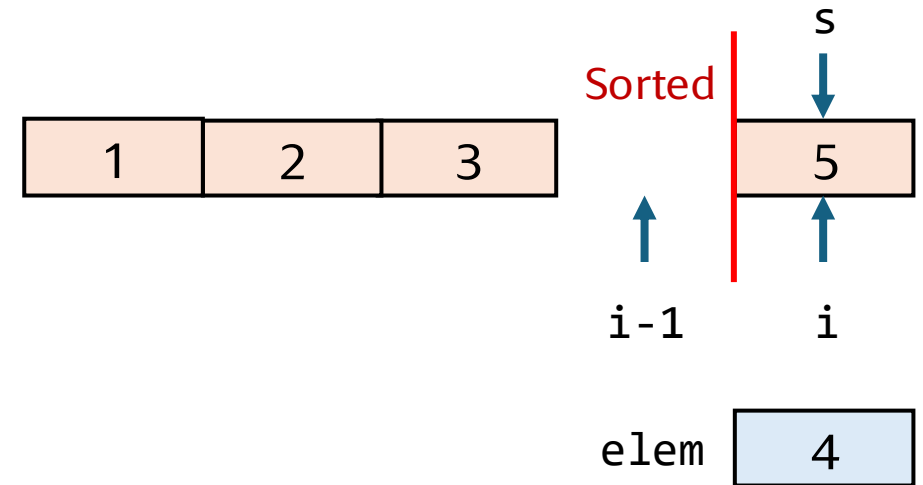
```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



`s = 4; i = 4; elem = 4`

# Insertion Sort with Moving

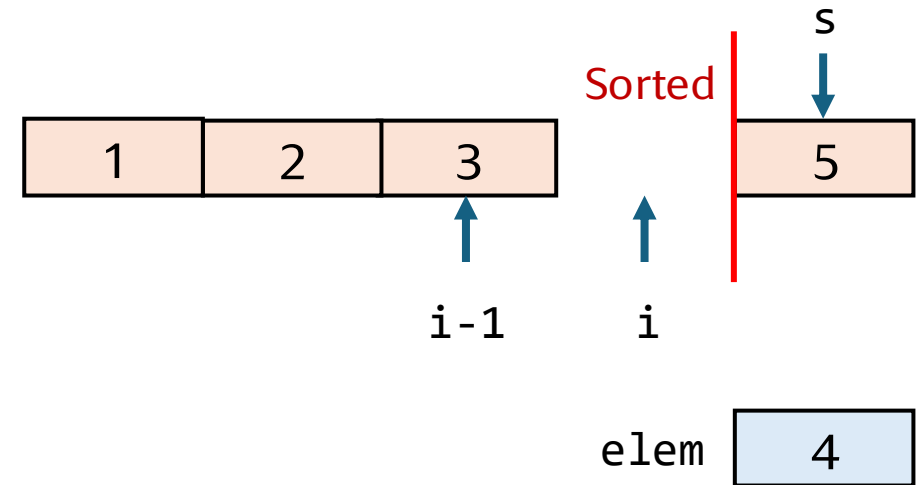
```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



`s = 4; i = 4; elem = 4`

# Insertion Sort with Moving

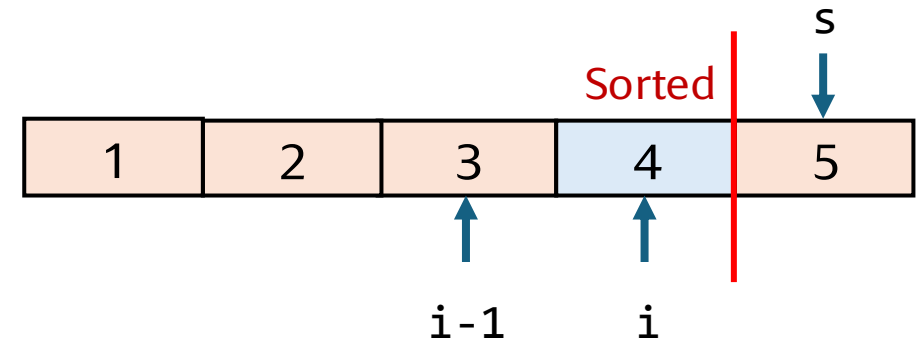
```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



`s = 4; i = 3; elem = 4`

# Insertion Sort with Moving

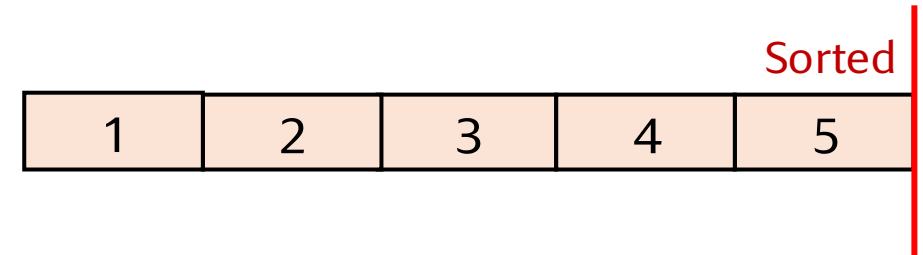
```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



`s = 4; i = 3; elem = 4`

# Insertion Sort with Moving

```
void insertion_sort_v1(int *arr, size_t sz) {  
    if (sz <= 1) return;  
  
    for (size_t s = 1; s < sz; s++) {  
        size_t i = s;  
        int elem = arr[s];  
        while (i > 0 && elem < arr[i - 1]) {  
            arr[i] = arr[i - 1];  
            i -= 1;  
        }  
        arr[i] = elem;  
    }  
}
```



Worst-case:  $n(n - 1)/2$  (comparisons + **moves**)

Best-case:  $(n - 1)$  comparisons

# Swapping vs. Moving

## Swapping

```
int tmp = arr[i];    // READ #1
arr[i] = arr[i - 1]; // READ #2, WRITE #1
arr[i-1] = tmp;     // WRITE #2
```

$n(n - 1)/2$  (comparisons + **swaps**)

2 reads

2 reads + 2 writes

$$2n(n - 1) \text{ reads} + n(n - 1) \text{ writes} \\ = (2 \text{ reads} + \text{write}) n^2 - (2 \text{ reads} + \text{write}) n$$

## Moving

```
arr[i] = arr[i - 1]; // READ #1, WRITE #1
```

$n(n - 1)/2$  (comparisons + **moves**)

2 reads

1 read + 1 write

$$1.5n(n - 1) \text{ reads} + 0.5n(n - 1) \text{ writes} \\ = (1.5 \text{ reads} + 0.5 \text{ writes}) n^2 - (1.5 \text{ reads} + 0.5 \text{ writes}) n$$

# Quick Sort

5	3	2	1	6	4	7
---	---	---	---	---	---	---

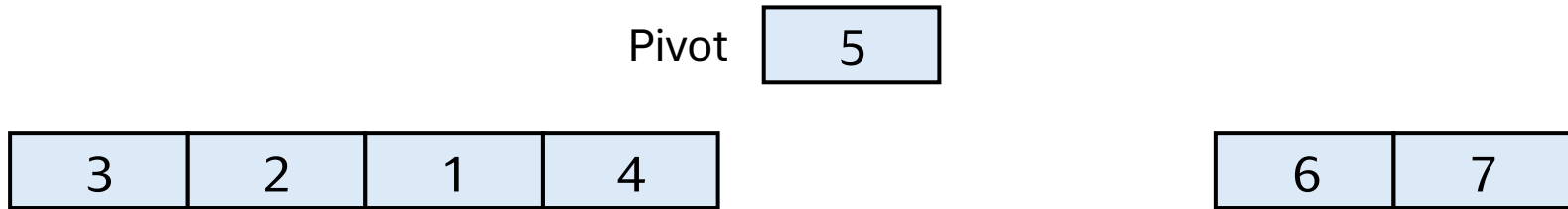
# Quick Sort



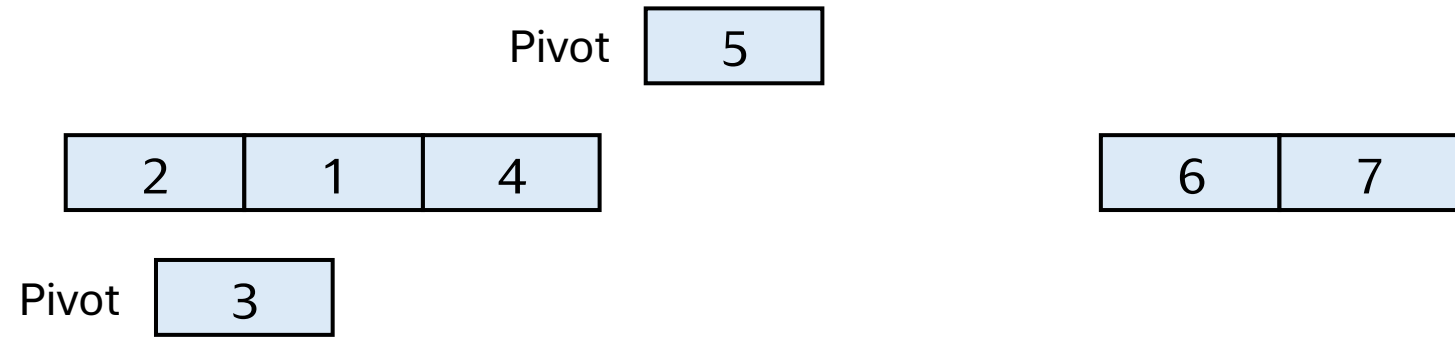
Pivot 

5
---

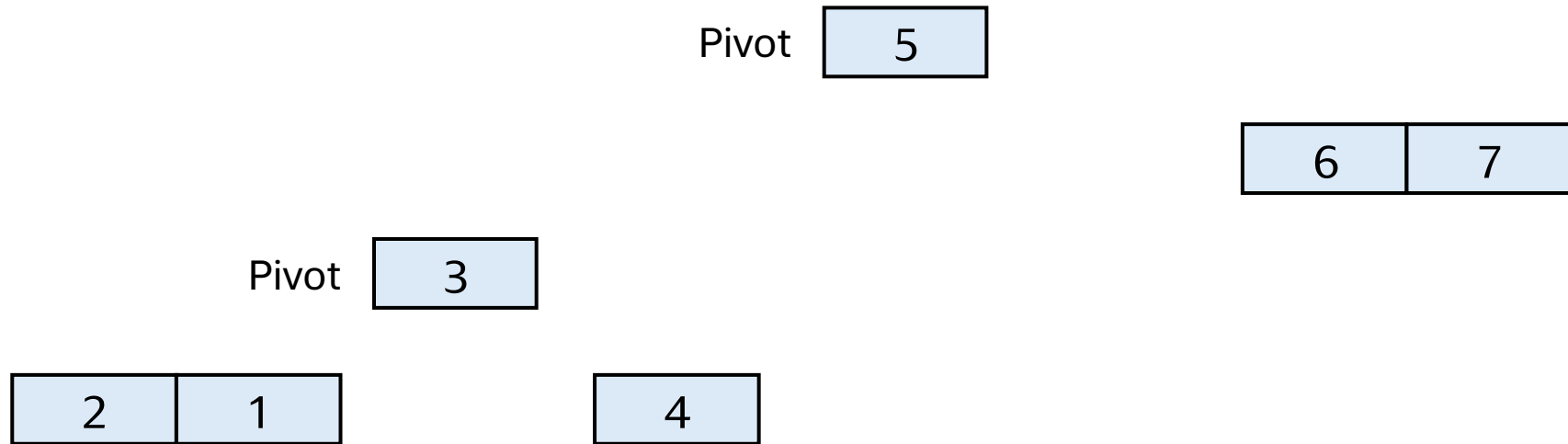
# Quick Sort



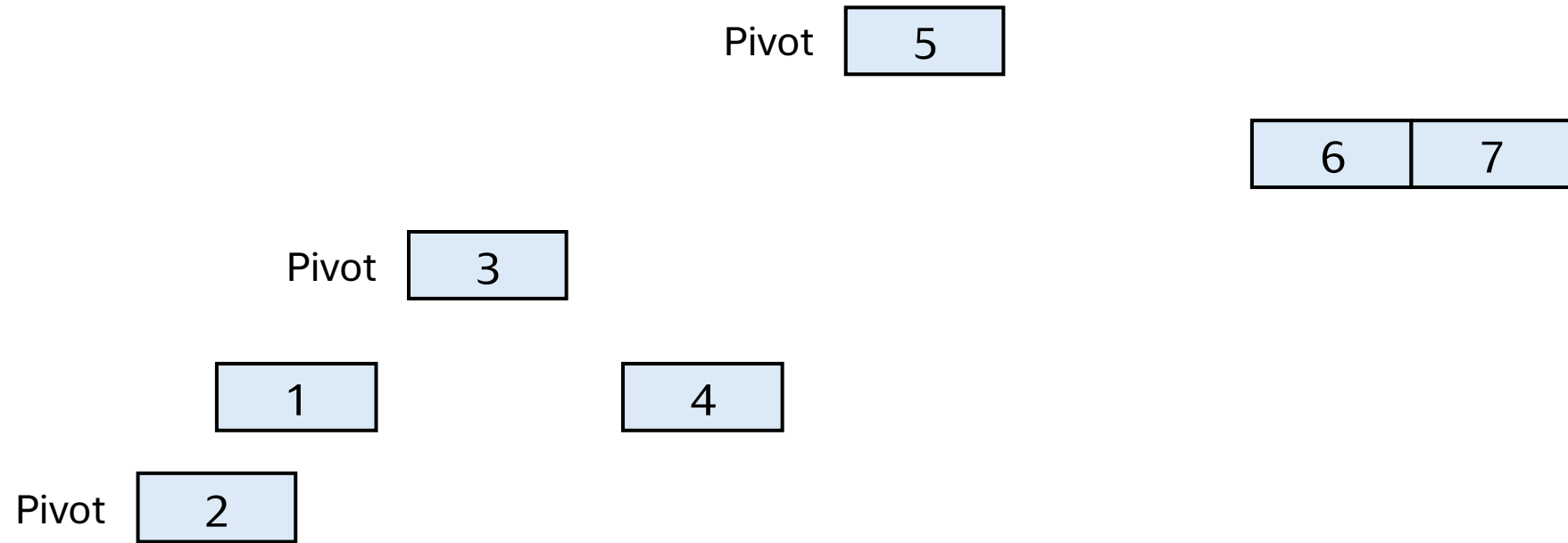
# Quick Sort



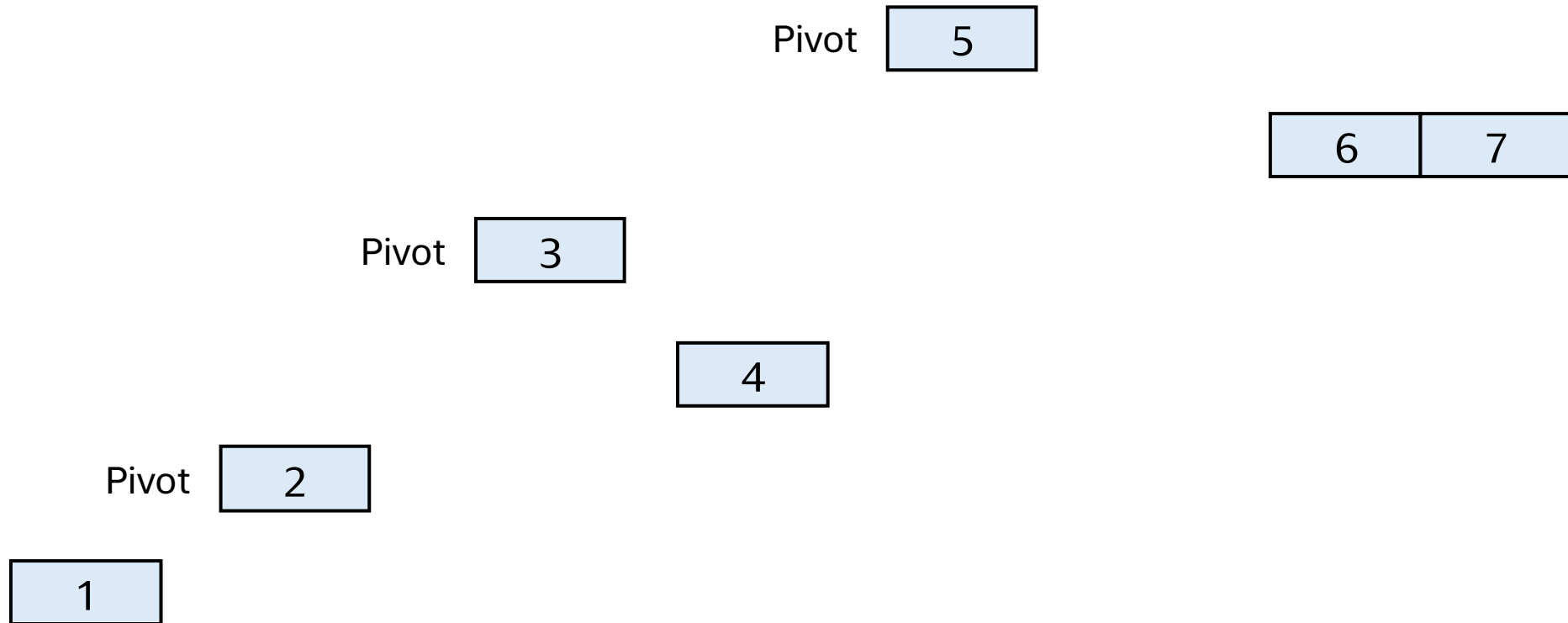
# Quick Sort



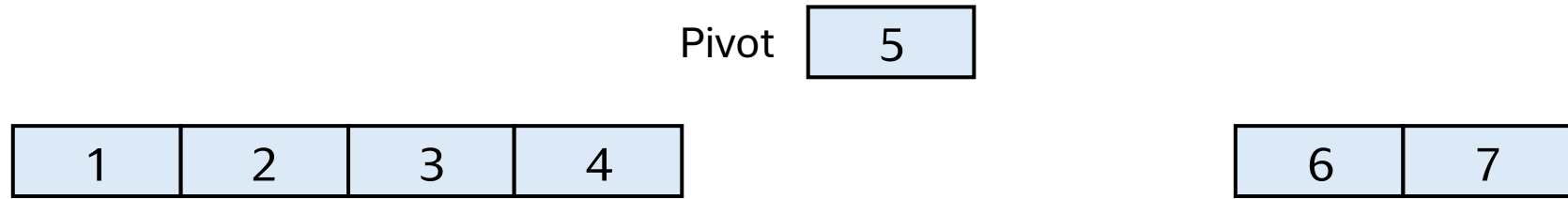
# Quick Sort



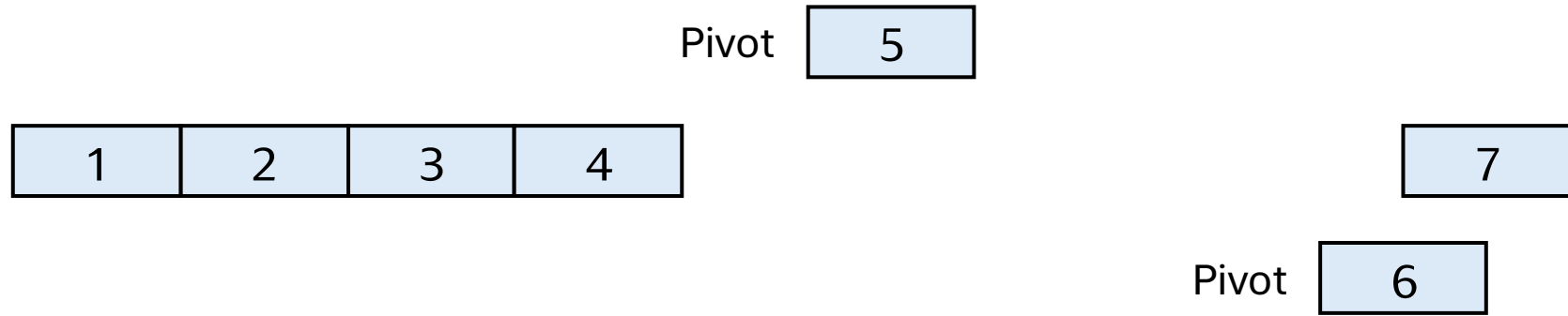
# Quick Sort



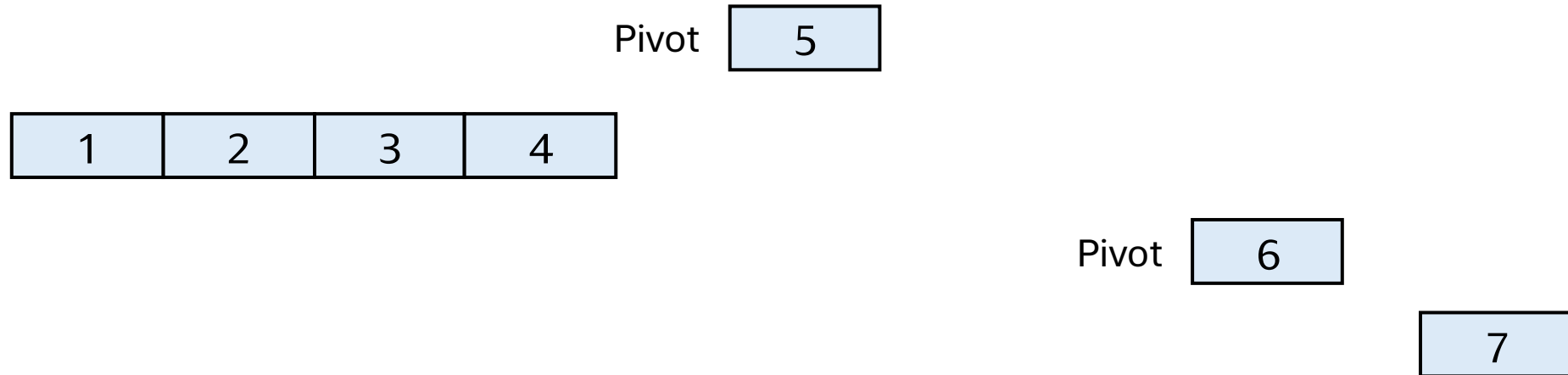
# Quick Sort



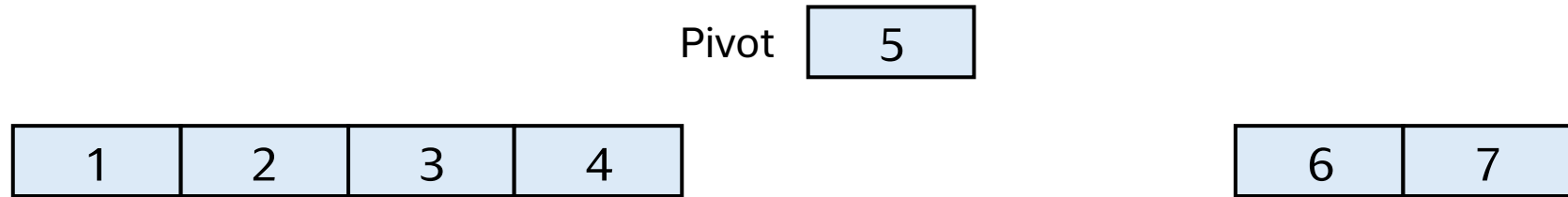
# Quick Sort



# Quick Sort



# Quick Sort



# Quick Sort

1	2	3	4	5	6	7
---	---	---	---	---	---	---

# Quick Sort

